ISSN (Online): 2455-3662
EPRA International Journal of Multidisciplinary Research (IJMR) - Peer Reviewed Journal
Volume: 6 | Issue: 10 | October 2020 || Journal DOI: 10.36713/epra2013 || SJIF Impact Factor: 7.032 ||ISI Value: 1.188

THE NUMERICAL SOLUTION BY THE METHOD OF DIRECT INTEGRALS OF DIFFERENTIATION OF EQUATIONS HAVE AN APPLICATION IN THE GAS FILTRATION THEOREM

Abdujabbor Abdurazakov
Candidate of Physics Sciences,
Associate Professor,
Ferghana Polytechnic Institute,
Uzbekistan,
Ferghana

Nasiba Makhmudova
Senior Lecturer,
Ferghana Polytechnic Institute,
Uzbekistan,
Ferghana

Nilufar Mirzamakhmudova
Senior Lecturer,
Ferghana Polytechnic Institute,
Uzbekistan,
Ferghana

Article DOI: https://doi.org/10.36713/epra5368

ANNOTATION

On the basis of the direct method and a combination of differential sweep, the article developed a calculated algorithm for solving gas filtration, thereby taking into account the convergence of the approximate solution to the exact one.

KEYWORDS: ***direct method, sweep method, differential equation, time step, convergence, approximate solution, error estimate.***

ANALYSIS

The problems of non-stationary filtering are of theoretical and practical interest [1]. Consider gas filtration taking into account pressure and velocity relaxation

The problem is to find in the region $\overline{\Omega}=\{0\le x\le 1,\ 0\le t\le T\}$ of a continuous function $u(x,t)$ satisfying in the equation

$$\frac{1}{m(x)}\frac{\partial}{\partial x}\left(k(x)\frac{\partial u}{\partial x}\right)=M(x,t,u)\frac{\partial u}{\partial t}+f(x,t,u)+\int_0^t R(t,s)ds \qquad (1)$$

Initial condition

$$u(x,0)=\varphi(x),\quad x\in[0,1], \qquad (2)$$

The boundary conditions are chosen depending on the convergence of the integrals

$$\int_0^1 \frac{dx}{k(x)} \text{ and } \int_0^1 \frac{\int_0^x m(\xi)d\xi}{k(x)}dx$$

If $\int_0^1 \frac{dx}{k(x)} < +\infty$, then

$$k(x)\frac{\partial u}{\partial x}\Big|_{x=0} = k(x)\frac{\partial u}{\partial x}\Big|_{x=1} = 0 \qquad (3)$$

If $\int_0^1 \frac{dx}{k(x)} = +\infty, \quad \int_0^1 \frac{\int_0^x m(\xi)d\xi}{k(x)}dx < +\infty$, then the conditions for $x=0$ are replaced by the condition

$$\left| u(x,0)\big|_{x=0} \right| < +\infty \qquad (4)$$

Here $k(x),\ m(x),\ f(x,t,u),\ M(x,t,u), R(t,s)$ -the given functions in the field of changing their arguments, $k(0)=0,\ k(x)$ and $m(x)$ moreover, are positive for $x>0,\ M(x,t,u)\geq m_0$ in the field $\{0\leq x\leq 1, 0\leq t\leq T, |u|<+\infty\}$.

We assume that all known functions in the $\overline{\Omega}$ equation are sufficiently $t=t_j$ smooth $t_j = j\tau,\quad j=1,\ldots,N,\ N=\left[\frac{T}{\tau}\right]$

We denote by the $u_j(x)$ approximate value of the desired function on the line $t=t_j$. We approximate the problems by the following scheme

$$\frac{1}{m(x)}\frac{d}{dx}\left(k(x)\frac{du_j}{dx}\right) = M\left(x,t_j,u_{j-1}\right)\delta_t u_j + f\left(x,t_j,u_{j-1}\right) + \tau\sum_{i=0}^{j-1} R_{j,i}u_i, \qquad (5)$$
$$j=\overline{1,n},$$

$$u_0(x) = \varphi(x)$$

If $\int_0^1 \frac{dx}{k(x)} < +\infty$, then the boundary conditions

$$k(x)\frac{du_j}{dx}\bigg|_{x=0} = k(x)\frac{\partial u_j}{\partial x}\bigg|_{x=1} = 0 \qquad j=\overline{1,n} \tag{6}$$

And if $\int_0^1 \frac{dx}{k(x)} = +\infty, \quad \int_0^1 \frac{\int_0^x m(\xi)d\xi}{k(x)}dx < +\infty$

then the conditions for $x=0$ replaced by conditions

$$\left\| u_j(x) \right\|_{x=0} < +\infty, \quad j=\overline{1,n} \tag{7}$$

Where

$$\delta_t - u_j = \frac{u_j - u_{j-1}}{\tau}, \quad j=\overline{1,N}$$

Problem (1) - (7) is solved sequentially from layer to layer starting $j=1$, and each time there is a unique solution corresponding to the boundary value problem (1) - (2) [1].

Estimating the solutions to problem (1) - (7), we obtain

$$\left\| u_j(x) \right\| \le \left| \frac{\frac{-M(x,t_j,u_{j-1})}{\tau} u_{j-1} + \tau \sum_{i=0}^{j-1} R_{j,i} u_i + f(x,t_j,u_{j-1})}{\frac{-M(x,t_j,u_{j-1})}{\tau}} \right| \le (1 + c_2 T\tau) \|u\|_{j-1} + c_1\tau, \quad j=\overline{1,n}$$

Hence

$$\|u\|_j \le (1 + c_2 T\tau)\|u\|_{j-1} + c_1\tau, \qquad j=1,\dots,N$$

where

$$\|u\|_j = \max_{1\le k\le i} |u_k|; \quad \|\circ\| = \max|\circ|, \qquad j=1,\dots,N$$

then easy to get $\|u\|_N \le \|\varphi\| e^{c_2T^2} + \frac{c_1}{Tc_2}\left(e^{c_1T^2} - 1\right)$

and also $\|u_j\| \le \|\varphi\| e^{c_2T^2} + \frac{c_1}{Tc_2}\left(e^{c_1T^2} - 1\right)$ for all $j=1,\dots,N$

Where the constants and - depend only on the given functions. The estimate is based on the maximum principle [1], [3].

ISSN (Online): 2455-3662
EPRA International Journal of Multidisciplinary Research (IJMR) - Peer Reviewed Journal
Volume: 6 | Issue: 10 | October 2020 || Journal DOI: 10.36713/epra2013 || SJIF Impact Factor: 7.032 ||ISI Value: 1.188

Similarly, we prove the uniform boundedness of the following quantities.

$$\left|\delta_{\bar{t}} u_j\right|, \left|k(x)\frac{du_j}{dx}\right|, \left|\frac{1}{m(x)}\frac{d}{dx}k(x)\frac{du_j}{dx}\right|, \left|\delta_{\bar{t}}\left(\delta_{\bar{t}} u_{j-1}\right)\right|, \left|k(x)\frac{d\phi_j}{dx}\right|, \left|\frac{1}{m(x)}\frac{d}{dx}k(x)\frac{d\phi_j}{dx}\right|$$

for all $j=1,...,10,\ \phi_j=\delta_{\bar{t}} u_j$

Uniform limited functions $\left|\frac{du_j}{dx}\right|, \left|\frac{d\phi_j}{dx}\right|$ depending on $\lim_{x\to 0}\int_0^x \frac{dx}{k(x)}$

Let $\lim_{x\to 0}\int_0^x \frac{dx}{k(x)}$ it exist and be finite.

We write analytically the linear extension formula

$$E^\tau(x,t)=\frac{t-t_{j-1}}{\tau}u_j(x)+\frac{t_j-t}{\tau}u_{j-1},\ \ j=1,...,N$$

We construct functions $u^\tau(x,t),\ u_t^{\ \tau}(x,t),\ k(x)u_x^{\ \tau},\ \frac{1}{m(x)}\frac{\partial}{\partial x}k(x)\frac{\partial u^\tau}{\partial x}$ using linear extension for $t\in\left[t_{j-1};t_j\right],\ j=\overline{1,N}$

The resulting family depends on the way the segment is split $[0,T]$.

The estimates obtained Ω imply uniform roundedness and equidistant continuity in, a family of functions $u^\tau(x,t),\ u_t^{\ \tau}(x,t),\ k(x)u_x^{\ \tau}$

These families are compact in uniform convergence. Therefore, it is possible to choose a sequence $\{\tau_j\}$ such that $\tau_j\to 0$, and the sequence $\{u^{\tau_j}\},\ \{u_t^{\ \tau_j}\},\ \{k(x)u_x^{\ \tau_j}\}$ converges uniformly in Ω and it follows that the sequence $\{u^{\tau_j}\}$ converges equally in the region $\Omega_\delta=\{\delta\le x\le 1,\ 0\le t\le\tau\}$ where $0\le\delta\le 1$. Due to randomness δ, we conclude that, $\{u_x^{\ \tau_j}\}$ converges at $\tau_j\to 0$ at each point $\Omega_\delta=\{\delta\le x\le 1,\ 0\le t\le T\}$.

In view of the linear extension formula, we have

$$\frac{1}{m(x)}\left(k(x)u_x^\tau\right)_x^1-M\left(x,t,u^\tau\right)u_t^\tau-f\left(x,t,u^\tau\right)-\int_0^1 R(t,s)u^\tau(x,s)ds=\varepsilon(\tau)$$

$$k(x)u_x^\tau\Big|_{x=0}=k(x)u_x^\tau\Big|_{x=1}=0$$

Where $\varepsilon(\tau)\to 0$ in $\tau\to 0$.

Passing to the limit in the chosen sequence, which $u(x,t)$ satisfies Ω Eq. (1) and with condition (2), (3).

ISSN (Online): 2455-3662
EPRA International Journal of Multidisciplinary Research (IJMR) - Peer Reviewed Journal
Volume: 6 | Issue: 10 | October 2020 || Journal DOI: 10.36713/epra2013 || SJIF Impact Factor: 7.032 ||ISI Value: 1.188

Suppose $\lim\limits_{x\to+0}\int\limits_0^x \frac{du}{k(u)} = +\infty$, then it can easily be established that

$\left|u^\tau(x'',t'') - u^\tau(x',t')\right| \le c_1\left|\sigma(x'') - \sigma(x')\right| + \mu_0\left(t'' - t'\right)$ where c_1, μ_1 -is some constant.

Here $\sigma(x) = \int\limits_0^x \frac{\int\limits_0^\xi m(\eta)d\eta}{k(\xi)} d\xi$, an increasing absolutely continuous function in [0,1].

Reasoning as in the proof $\int\limits_0^1 \frac{dx}{k(x)} < +\infty$, we come to the assertion that in the domain Ω there exists a solution to equation (1) satisfying the initial conditions (2) and boundary by conditions (3) - (4).

The numerical implementation of the solution of problems (5) - (6) will use the modified sweep method [1],

Direct sweep: to construct a numerical solution $\alpha_j(x),\ \beta_j(x)$ in the field $\{0 \le x \le \delta\}$, $\delta -$ of a sufficiently small number, by the formulas

$$\alpha_j(x) = \frac{1}{V_j(x)}\left(1 + \int\limits_0^x m(\xi)\frac{M(\xi,t_j,u_{j-1}}{\tau}V_j(\xi)d\xi\right),$$

$$\beta_j(x) = \frac{1}{V_j(x)}\left(1 + \int\limits_0^x \left(m(\xi)\frac{-M(\xi,t_j,u_{j-1})}{\tau}u_{j-1} + \tau\sum_{i=0}^{j-1}R_{i,j}u_i + f(\xi,t_j,u_{j-1})\right)d\xi\right)$$

where $V_j(x) = 1 + \int\limits_0^x \frac{\int\limits_0^h m(\xi)\frac{M\left(\xi,t_j,u_{j-1}\right)}{\tau}d\xi}{k(h)}dh$

We seek the solution of integral equations in the form of a series.

$$V_j(x) = \sum_{j=0}^{\infty}\sigma_j(x), \qquad j = \overline{1,N}$$

$\sum\limits_{j=0}^{\infty}\sigma_j(x)$ - the series converges uniformly.

By the method of successive approximations, the terms of the series are determined by the following relations

$$\sigma_0 = 1,\ \ \sigma_j(x) = 1 + \int\limits_0^x \frac{\int\limits_0^h m(\xi)\frac{M(\xi,t_j,u_{j-1})}{\tau}\sigma_{j,k-1}(\xi)d\xi}{k(h)}dh \qquad j = 1,2,\ldots,N$$

$\sigma_j(x)$ absolute continuous and monotonically increasing function. To calculate the integrals involved in the recurrence relations, the method of singling out features proposed by Kontorovich is used

After finding $\alpha_j(x)$ and $\beta_j(x)$, $j=\overline{1,N}$ on the interval $[\delta,1]$ using the Runge-Kutta method, we solve the system of equations

$$\begin{cases} \alpha_j^x(x) = m(t)\dfrac{M\left(x,t_j,u_{j-1}\right)}{\tau}u_{j-1}\dfrac{\alpha_j^2}{k(x)} \\ \beta_j^/(x) = \left[\dfrac{M\left(x,t_j,u_{j-1}\right)}{\tau} + \tau\sum_{j=1}^{j_2} R_j u_j + f\left(x,t_j,u_{j-1}\right)\right]m(x) - \dfrac{\alpha_j(x)\beta_j(x)}{k(x)} \end{cases}, \qquad j=\overline{1,N}$$

with initial condition

$$\alpha_j(x)\big|_{x=\delta} = \alpha_j(\delta)$$
$$\beta_j(x)\big|_{x=\delta} = \beta_j(\delta), \quad j=1,\dots,N$$

Reverse run:

We consider the equation in the form

$$\frac{du_j}{dx} = \frac{M(x,t_j,u_{j-1})u_j + \left(\dfrac{-M(x,t_j,u_{j-1})}{\tau}u_{j-1} + \tau\sum_{t=1}^{j-1} R_{i,j}u_i + f\left(x,t_j,u_{j-1}\right)\right)}{k(x)}$$

Under the initial condition

$$u_j(1) = -\frac{\beta_j(1)}{\alpha_j(1)}, \quad j=1,\dots,N$$

This equation has singularities for $x \to +0$

If $\lim\limits_{x\to+0}\int_0^x m(\varsigma)\dfrac{\mu(\xi,t_j,u_{j-1})}{k(x)}d\xi$ (*) exists, of course, it can be eliminated by calculating the limits $\lim\limits_{x\to+0}\dfrac{\alpha_j(x)}{k(x)}$ and $\lim\limits_{x\to+0}\dfrac{\beta_j(x)}{k(x)}$ of $j=1,\dots,N$. Eliminating these features, we find a solution according to the Runge-Kutta method for $j=1,\dots,N$

If it does not exist, then we first construct the solution of the equation in the region $\{\delta \le x \le 1\}$, according to the Runge-Kutta method

Then using the built.

$$U_j(x)=\left(\frac{u_j(\delta)}{V_j(\delta)}-\int_x^{\delta}\frac{\left(\frac{-M(x,t_j,u_{j-1})}{\tau}u_{j-1}+\tau\sum_{t=1}^{j-1}R_{i,j}u_i+f(x,t_j,u_{j-1}\right)}{k(\xi)V_j(\xi)}d\xi\right)V_j(x),\quad j=1,\dots,N$$

We find $u_j(x)$ in the area $[0,\delta]$, $j=1,\dots,N$

We propose one of the possible methods for the numerical solution of problem [1] and [6].

Note: An approximate solution constructed by the method of lines converges to an exact solution with a speed $0(\tau)$ where is a time step τ

LITERATURE

1. *Abdurazakov A. Sxodimost metoda pryamyx dlya integer-differentialnyx uravneniy.V sb.”Vychislitelnaya matematika I matematicheskaya statistika”,issue 8,1979, MGPX*
2. *Abdurazakov A, Makhmudova N, Mirzamakhmudova N. Solutions of a multi-point boundary value problem of gas filtration in multilayer layers with relaxation taken into account. Universum: technical sciences: electronic scientific journal 2019 №11 (68) p5-9*
3. *Bobkov V.V. Liskovets O.A. Point estimates in the Rothe method. Differential Equations 2, 1966, No. 5 p 140-640*