EXPERIMENTAL DETERMINATION OF TEMPERATURE PROFILES IN UNSTEADY STATE IN A LABORATORY OF TRANSPORT PHENOMENA.

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ABSTRACT
In the laboratory of Transport Phenomena, the students of Chemical Engineering at the Faculty of Chemistry of the UNAM, in Mexico, carry out experiments in Transport Phenomena. This time an experiment is presented to obtain the temperature profiles and the thermal conductivity in spheres in unsteady state.

KEY WORDS: Transport Phenomena, heat transfer, unsteady state, laboratory practices.

1.-INTRODUCTION
The heat transfer in unsteady state is an operation that appears in a variety of transformation industries. In the undergraduate classes, this phenomenon is generally presented in a theoretical way, so it is necessary that the students see the phenomenon in the laboratory so that they can understand and evaluate it. There are different ways to experimentally obtain the data, but in this journal, we present a method to obtain them by applying the second Fourier’s law which is used to treat the heat transfer in unsteady state.

1.1.- Second Fourier’s law
The second Fourier’s law for heat transfer by conduction can be expressed as [1, 2, 3, 4]:

\[
\frac{\partial T}{\partial \theta} = \alpha \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] \tag{1}
\]

Or in abbreviated form:

\[
\frac{\partial T}{\partial \theta} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{2}
\]

Where the term \(\alpha\) represents the thermal diffusivity defined as:

\[
\alpha = \frac{k}{\rho C_p} \tag{3}
\]

Where \(k\) is the thermal conductivity of the material, \(\rho\) is the density and \(C_p\) is the heat capacity of the material.

By means of dimensional analysis it can be deduced that the solutions for the equations at transitional regime are in function of the following dimensionless numbers [4, 5]:

Temperature change module:

\[
\frac{T_s - T}{T_s - T_0} \tag{4}
\]
Module or Fourier number:
\[ F_0 = \frac{a \theta}{x_1^2} \]  
(5)

Module or Biot number:
\[ Bi = \frac{x_1 h}{k} \]  
(6)

The Biot number measures the relative importance of the external resistance to the internal resistance.

Position module.
\[ \frac{x}{x_1} \]  
(7)

Where \( T_s \) is the temperature on the surface, \( T_0 \) is the initial temperature and \( T \) is the temperature at any time. \( \alpha \) is the thermal diffusivity, \( \theta \) is the time, \( x_1 \) is the maximum distance that heat must travel through the body, \( x \) is the position where is required obtain the temperature, \( h \) is the heat transfer coefficient and \( k \) is the thermal conductivity.

1.1. Heating or cooling of a specimen with high thermal conductivity

If the thermal conductivity is high, the internal resistance to energy transfer can be considered negligible. This limiting situation is more easily obtainable if the specimen has a large surface area compared to its volume. A process in which the internal resistance is ignored, and the energy transfer process is expressed in terms of the surface controlling resistance is known as the Newtonian heating or cooling process. For this case, the solution for the heat transfer equation in unsteady state is \([1, 2, 3,4]\):
\[ \frac{T - T_a}{T_0 - T_a} = e^{-\frac{hA\theta}{\rho c pV}} \]  
(8)

Where \( T_a \) is the room temperature, \( A \) is the area and \( V \) is the specimen volume. This is true if \( Bi < 0.1 \).

1.2. Heating of a specimen with negligible surface resistance

For these cases, the surface temperature \( T_s \) is constant with respect to time and its value is essentially equal to room temperature \( T_a \).

For this case, the solution for a sphere is of the type \([1, 2, 3,4]\):
\[ \sum_{i=1}^{\infty} \left( \frac{i\pi}{R} \right)^2 \cos \left( \frac{i\pi}{R} \right) \exp \left( -\frac{i^2 \pi^2 \alpha \theta}{R^2} \right) \]  
(9)

Where \( R \) is the radius of the sphere and \( r \) is any radial position. For the sphere center the above equation is reduced to:
\[ Y = -2 \sum_{i=1}^{\infty} \left( \frac{i\pi}{R} \right)^2 \cos \left( \frac{i\pi}{R} \right) \exp \left( -\frac{i^2 \pi^2 \alpha \theta}{R^2} \right) \]  
(10)

The solutions for this kind of problems are usually presented in graph form, such as those of Gurney-Lurie \([3,4,5,7]\).

2. PROPOSED PRACTICES

The proposed practices allow the student to demonstrate the effect of the unsteady state conduction by using spherical geometries for wood and aluminum which are submerged in a thermostatic bath. For this, spheres of known diameters and different materials are initially found at room temperature. Each one has a detachable thermocouple in its center. Suddenly the spheres are submerged in a water bath at a higher temperature than room temperature, producing a temperature gradient in the materials. By the thermocouple readings it is possible follow the temperature history.

2.1. Required material
- Type K thermocouple
- Stopwatch
- Spheres
- Thermostatic bath
Figure 1.- Equipment used

<table>
<thead>
<tr>
<th></th>
<th>Aluminum sphere</th>
<th>Wooden sphere</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Diameter (cm)</strong></td>
<td>5.9</td>
<td>10</td>
</tr>
<tr>
<td><strong>Heat capacity, (C_p), (J/kg k)</strong></td>
<td>903</td>
<td>1380</td>
</tr>
<tr>
<td><strong>Density, (\rho), (kg/m(^3))</strong></td>
<td>2702</td>
<td>510</td>
</tr>
<tr>
<td><strong>Thermal conductivity, (k), (W/mK)</strong></td>
<td>237</td>
<td></td>
</tr>
</tbody>
</table>

2.2.- **Experimental technique** [7]

- a) The student will assemble the equipment as shown in the figure 1.
- b) The thermostatic bath must be filled by feeding water with the inlet hose up to the mark.
- c) Connect the inlet hose to recirculation system.
- d) Turn on the thermostatic bath in order to recirculate the liquid inside this.
- e) Set the maximum temperature for the bath and let the bath reach the desired temperature.
- f) Submerge the supports for each sphere first.
- g) Submerge the spheres. These should relate to the thermocouple in the center (make sure to fit the plug in the hole where the thermocouple is placed before submerging them).
- h) Proceed to read the temperature variations with respect to time.
- i) Once the permanent regime has been reached, remove the spheres.
- j) Turn off the circulation and the heating of thermostatic bath.
- k) Finish by emptying the water tank.
2.3 - Experimental data
In the experiments the room temperature was 21°C and the maximum temperature in the bath was 50°C. The students introduced the aluminum and wooden spheres in the bath and the following data were obtained:
### Table 1

<table>
<thead>
<tr>
<th>Time in seconds</th>
<th>Temperature at the center in °C</th>
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<tbody>
<tr>
<td>0</td>
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<tr>
<td>10</td>
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<td>90</td>
<td>50</td>
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### Table 2

<table>
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<th>Time in minutes</th>
<th>Temperature at the center in °C</th>
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</thead>
<tbody>
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<td>22</td>
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<tr>
<td>0.5</td>
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<tr>
<td>1</td>
<td>24</td>
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<tr>
<td>1.5</td>
<td>24</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
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<td>2.5</td>
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<td>25</td>
<td>46</td>
</tr>
<tr>
<td>30</td>
<td>47</td>
</tr>
</tbody>
</table>
3.- DATA TREATMENT

From the data obtained the students generated the following graphs:

**Runs for aluminum sphere at 50°C**

Graph 1. - Behavior of aluminum sphere
Runs 1, 2 & 3 for aluminum sphere at 50°C

**Graph 2. - Heating of wooden sphere**
Runs 1, 2 & 3 for aluminum sphere at 50°C
3.1.- Heating of an aluminum sphere.
This case corresponds to that of a specimen with high thermal conductivity and can be evaluated by Newton’s equation:

\[ \frac{T - T_a}{T_0 - T_a} = e^{-\frac{hA_0}{\rho c_p V}} \]  

For example, for aluminum with a time of heating of 30 seconds, a temperature in the center of the aluminum sphere of 42°C was obtained. The bath temperature was 50°C and the initial temperature of the sphere was 23°C. The sphere volume is:

\[ V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (0.0295)^3 = 1.075 \times 10^{-4} \text{ m}^3 \]

The sphere area is:

\[ A = 4\pi r^2 = 4\pi (0.0295)^2 = 0.01093 \text{ m}^2 \]

Thus:

\[ \frac{50 - 42}{50 - 23} = \exp \left( -\frac{h(0.01093)(30)}{2702(903)(1.075 \times 10^{-4})} \right) \]

And clearing, \( h = 1012 \text{ W/m}^2\text{K} \)

Carrying out the same calculations for the whole run and taking an average, \( h = 1032 \text{ W/m}^2\text{K} \) is obtained. With this data is possible to draw the following graph where the comparison of the theoretical data with the experimental data is shown.

3.2.- Heating of wooden sphere
In the experiment, data was obtained in the center of the wooden sphere, so the equation applicable in this case was:

\[ Y = -2 \sum_{i=1}^{\infty} (-1)^i \exp \left( -\frac{i^2 \pi^2 \alpha t}{R^2} \right) \]  

For \( i = 1 \)

The equation is reduced to:

\[ Y = -2 \exp \left( -\frac{\pi^2 \alpha t}{R^2} \right) \]

From there:

\[ \alpha = -\left( \frac{R^2}{\pi^2 \theta} \right) \ln \frac{Y}{2} \]

For the time equal to 23 minutes it was found that \( T = 45 \)°C, being the initial temperature of the sphere of 22°C and that of the bath of 50°C. The radius of the sphere was 0.05 m.

The sphere area is:

\[ A = 4\pi r^2 = 4\pi (0.0295)^2 = 0.01093 \text{ m}^2 \]

Thus:

\[ \frac{50 - 42}{50 - 23} = \exp \left( \frac{50 - 22}{2702(903)(1.075 \times 10^{-4})} \right) \]

And clearing, \( h = 1012 \text{ W/m}^2\text{K} \)

Carrying out the same calculations for the whole run and taking an average, \( h = 1032 \text{ W/m}^2\text{K} \) is obtained. With this data is possible to draw the following graph where the comparison of the theoretical data with the experimental data is shown.
Graph 4.-Behavior of the wood sphere

From the experimental data, the thermal conductivity of the wood sphere is:

\[ k = \alpha \rho C_p = 4.12 \times 10^{-7} \frac{m^2}{s} \times 510 \frac{kg}{m^3} \times 1380 \frac{J}{smK} = 0.2899 \frac{W}{mK} \]

4.- CONCLUSION

By means of a simple experiment carried out in a non-complex equipment, students of Transport Phenomena, at Faculty of Chemistry, UNAM, were able to examine the heat transfer phenomenon in unsteady state in spheres; one with great thermal conductivity and the other with a small thermal conductivity. The results obtained experimentally by the students agree quite well with those predicted by the theory.

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