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FUZZY MULTIOBJECTIVE GOAL PROGRAMMING IN OPTIMIZATION PRODUCTION FORMATION

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ABSTRACT

The solution of fuzzy (weighted) goal programming FGP problem is solved by a new method by using simplex method. Here, the relative weights indicate the absolute significance of the objective functions. This method consists of supplementary goal constraint and deviation variables to the fuzzy problem which is dissimilar from other methods. In addition, the proposed methods are easy to apply in real-life situations which give better solution in the sense that the objective values are sufficiently closer to their aspiration levels. Finally, for illustration, a real-life example is used to demonstrate the proposed methods.

KEYWORDS: fuzzy, programming, optimization, decision makers

INTRODUCTION TO GOAL PROGRAMMING

Goal programming can be considered as a branch of multi-objective optimization. Goal programming is one of the oldest multi-criteria decision making techniques used in optimization of multiple objective goals by minimizing the deviation for each of the objectives from the desired target. The basic concept of goal programming is that whether goals are attainable or not an objective will be stated in which optimization gives a result which come as close as possible to the desired goals. The objective of goal programming is to minimize the non-achievement of each goal level. If non-achievement is driven to zero, then it means that actual attainment of the goal has been accomplished. For a single goal problem, the formulation and solution is similar to linear programming with the exception that, if complete goal attainment is not possible, goal programming will provide a solution and information to the decision makers.
GENERAL MODEL

Min \( Z = \sum_{i=1}^{m} w_i p_i (d_i^- + d_i^+) \)
subject to \( \sum_{i=1}^{m} a_{ij} x_j + d_i^- - d_i^+ = b_i \)
\( i = 1, 2, ..., m \) and \( x_j, d_i^-, d_i^+ \geq 0; j = 1, 2, ..., n \) and \( d_i^+ x d_i^- = 0 \)

where there are \( m \) goals, \( P \) system constraints and \( n \) decision variables

\( Z = \) objective function = Summation of all deviations
\( a_{ij} = \) the coefficient associated with variable \( j \) in the \( i^{th} \) goal
\( x_j = \) the \( j^{th} \) decision variable
\( b_i = \) the associated right hand side value, which is a fuzzy value.
\( d_i^- = \) negative deviational variable from the \( i^{th} \) goal (underachievement)
\( d_i^+ = \) positive deviational variable from the \( i^{th} \) goal (overachievement).

Goal programming may be used to solve linear programs with multiple objectives, with each objective viewed as a "goal". In goal programming, \( d_i^- \) and \( d_i^+ \), deviation variables, are the amounts a targeted goal \( i \) is overachieved or underachieved, respectively. The goals themselves are added to the constraint set with \( d_i^- \) and \( d_i^+ \) acting as the surplus and slack variables. One approach to goal programming is to satisfy goals in a fuzzy priority sequence. Second-priority goals are not pursued without reducing the first-priority goals, etc. For each fuzzy priority level, the objective function is to minimize the (weighted) sum of the goal deviations.

FORMULATION OF GOAL PROGRAMMING MODEL

Step 1: Decide the fuzzy priority level of each goal with fuzzy priority level \( P_1 \) as most important, followed by \( P_2 \) and so on.

Step 2: Decide the weight on each goal. If a fuzzy priority level has more than one goal, for each goal \( i \) decide the weight, \( w_i \), to be placed on the deviation(s), \( d_i^- \) and/or \( d_i^+ \); from the goal.

Step 3: Set up a linear program. Write the objective function in terms of minimizing a prioritized function of the deviational variables, subject to all Functional and Goal Constraints.

Step 4: Solve the current linear program using simplex method.

SIMPLEX METHOD OF FUZZY GOAL PROGRAMMING

Step 1: Convert the given LGP into standard form.

Step 2: Enter the data into a simplex table. Unlike linear programming the \( z_j - c_j \) is divided into as many numbers of rows as the number of priorities assigned. The \( z_j \) and \( z_j - c_j \) values are calculated separately for each of the ranked goal \( P_1, P_2, \ldots \). On the basis of the fuzzy priority, the first fuzzy priority goal \( P_1 \) is shown at the bottom and the least prioritized goal is shown at the top.

Step 3: If all \( z_j - c_j \leq 0 \) and if the target value of each goal in \( X_{ij} \) column is zero, then the current solution is optimal. Otherwise go to next step

Step 4: Select the most positive value at the highest fuzzy priority level. Here \( P_1 \gg P_2 \gg \ldots \gg P_n \) This corresponds to the pivot column. Now compute the ratio, \( \text{Min}(X_{ji}/a_{ij}, a_{ij} > 0) \). This corresponds to the pivot row and the intersection of the row and column is called the pivot element.

Step 5: prepare the simplex table for next iteration in the same manner as the usual simplex method and repeat the procedure from Step 3 until an optimal solution is obtained.

EXAMPLE

The company produces two types of bobbin holders: rotating type and non-rotating type. The moulding section can produce a maximum of 400-450 rotating type and 500-540 non rotating type bobbin holders in an hour. The profit contributions of each type of bobbin holder are Rs.22 and Rs.30 respectively. The management has prioritized the following goals:

\( P_1: \) to minimize the underachievement of obtaining a profit of \( \{23000, 25000, 28000\} \)
\( P_2: \) to minimize the underachievement of manufacturing utmost \( \{400, 440, 450\} \) product 1
\( 2P_2: \) to minimize the underachievement of manufacturing utmost \( \{506, 524, 535\} \) product 2

MATHEMATICAL FORMULATION

Decision variables:
\( x_1: \) No of pieces of bobbin holder type 1
\( x_2: \) No of pieces of bobbin holder type 2
\( d_i^-: \) underachievement associated with goal \( i \)
\( d_i^+: \) overachievement associated with goal \( i \)

Objective function:
The objective is to minimize the underachievement and overachievement of each goal on the basis of the fuzzy priority given.
Minimize \( Z = P_1 d_i^- + P_2 d_i^+ + 2P_2 d_i^- \)

Constraints:
The demand constraints are:
\( 22x_1 + 30x_2 + d_i^- - d_i^+ = \{23000, 25000, 28000\} \)
\( x_1 + d_i^- = \{400, 440, 450\} \)
\( x_2 + d_i^+ = \{506, 524, 535\} \)

Non-negativity restrictions:
\( x_1, x_2, d_i^-, d_i^+, d_i^- \geq 0 \)
**SOLUTION:**

**Step 1:** The initial basic solution is obtained by assigning \( n-m = 7-5 = 2 \) variables 0.

\[
\begin{align*}
& d_1^- = \{23000, 25000, 28000\} & d_2^- = \{400, 440, 450\} & d_3^- = \{506, 524, 535\} \\
& (x_1 = x_2 = d_1^- = 0 \text{ which are non basic variables})
\end{align*}
\]

**Step 2:** The simplex table is represented as

Initial Iteration:

<table>
<thead>
<tr>
<th>( C_B )</th>
<th>( B )</th>
<th>( X_B )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( d_1^- )</th>
<th>( d_2^- )</th>
<th>( d_3^- )</th>
<th>( d_1^+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 )</td>
<td>( d_1^- )</td>
<td>{23000, 25000, 28000}</td>
<td>22</td>
<td>30</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>( P_2 )</td>
<td>( d_2^- )</td>
<td>{400, 440, 450}</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( 2P_2 )</td>
<td>( d_3^- )</td>
<td>{506, 524, 535}</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( z_f-c_j )</td>
<td>( P_2 )</td>
<td>{1412, 1488, 1520}</td>
<td>22</td>
<td>30</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

Since not all \( z_f-c_j \leq 0 \), the current solution is not optimal. Since the most positive value occurs in the \( x_2 \) column it enters the basis \( \theta = \text{Minimum} \left\{ \frac{x_i}{a_{ik}}, a_{ik} > 0 \right\} \), \( \theta = 521.66 \) the corresponding non-basic variable \( d_2^- \) leaves the basis.

**Step 3:** first iteration

Similarly the First has been derived and the result interpreted as follows. Since not all \( z_f-c_j \leq 0 \), the current solution is not optimal. Since the most positive value occurs in the \( x_1 \) column it enters the basis \( \theta = \text{Minimum} \left\{ \frac{x_i}{a_{ik}}, a_{ik} > 0 \right\} \), \( \theta = 430 \), the corresponding non-basic variable \( d_1^- \) leaves the basis.

**Step 4:**

Second iteration

<table>
<thead>
<tr>
<th>( C_B )</th>
<th>( B )</th>
<th>( X_B )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( d_1^- )</th>
<th>( d_2^- )</th>
<th>( d_3^- )</th>
<th>( d_1^+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 )</td>
<td>( d_1^- )</td>
<td>{-980, -400, 2050}</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-22</td>
<td>-30</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>( x_1 )</td>
<td>{400, 440, 450}</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>( x_2 )</td>
<td>{506, 524, 535}</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( z_f-c_j )</td>
<td>( P_2 )</td>
<td>{0,0,0}</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>( P_1 )</td>
<td>{-980, -400, 2050}</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-22</td>
<td>-30</td>
<td>-1</td>
<td></td>
</tr>
</tbody>
</table>

Since all \( z_f-c_j \leq 0 \), the current solution is optimal. The optimal solution is \( x_1 = \{400, 440, 450\} \), \( x_2 = \{506, 524, 535\} \), \( d_1^- = \{-980, -400, 2050\} \), \( d_1^+ = 0 \), \( d_2^- = 0 \), \( d_1^- = 0 \).

**CONCLUSION**

In the decision-making problem, there may be situations where a decision maker has to content with a solution of the FGP problem where some of the fuzzy goals are achieved and some are not because these fuzzy goals are subject to the function of environment/resource constraints. Since the relative weights represent the relative importance of the objective functions, the proposed effective in finding the optimal solution or near optimal solution of the fuzzy goal programming problems and helps to achieve the goals completely.

**REFERENCES**