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ON THE PAIR OF EQUATIONS

$$a \pm b = p^3, ab = q^2$$

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ABSTRACT

This communication aims at determining pairs of non-zero distinct integers (a, b) such that, in each pair

- (i). *the sum is a cubic integer and the product is a square integer*
- (ii). *the difference is a cubical integer and the product is a square integer*

KEYWORDS: *system of double equations, integer solutions*

1. INTRODUCTION

In the history of number theory, the Diophantine equations occupy a remarkable position as it has an unlimited supply of fascinating and innovating problems [1-9]. This communication concerns with the problem of obtaining two non-zero distinct integers a and b such that

(i). $a + b = p^3, ab = q^2$ and

(ii). $a - b = p^3, ab = q^2$

2. METHOD OF ANALYSIS

(I) On the system $a + b = p^3, ab = q^2$

Let a, b be two non-zero distinct positive integers such that

$$a + b = p^3, ab = q^2 \tag{1,2}$$

where $p, q > 0$

The elimination of b between (1) and (2) leads to

$$a^2 - ap^3 + q^2 = 0$$

which is satisfied by

$$a = \frac{1}{2} \left(p^3 + \sqrt{p^6 - 4q^2} \right) \tag{3}$$

The square root on the RHS is eliminated when

$$q = rs, \quad p^3 = r^2 + s^2, \quad r > s > 0 \tag{4,5}$$

and thus, note that

$$a = r^2, \quad b = s^2 \tag{6}$$

Now, note that the values of r and s should satisfy (5). After some algebra, it is seen that there are two sets of values of r, s given as below:

$$\text{Set 1: } r = m(m^2 + n^2), \quad s = n(m^2 + n^2)$$

$$\text{Set 2: } r = m^3 - 3mn^2, \quad s = 3m^2n - n^3$$

where $m, n \neq 0$

Using Set 1, the values of a, b satisfying (1,2) are given by

$$a = m^2(m^2 + n^2)^2, \quad b = n^2(m^2 + n^2)^2$$

and in view of set 2, one has

$$a = (m^3 - 3mn^2)^2, \quad b = (3m^2n - n^3)^2$$

However, it is worth to mention that the square root on the RHS of (3) is also eliminated when

$$q = 2(r^2 - s^2), \quad p^3 = 4(r^2 + s^2) \tag{7,8}$$

and we obtain

$$a = 2(r + s)^2, \quad b = 2(r - s)^2 \tag{9}$$

Now, observe that r and s should satisfy (8). It is seen that there are two sets of values to r, s as presented below:

Set 3:

$$r = \alpha^3 - 3\alpha\beta^2 - 3\alpha^2\beta + \beta^3$$

$$s = \alpha^3 - 3\alpha\beta^2 + 3\alpha^2\beta - \beta^3$$

$$p = \alpha^2 + \beta^2$$

Set 4:

$$r = \frac{m(m^2 + n^2)}{2}$$

$$s = \frac{n(m^2 + n^2)}{2}$$

$$p = m^2 + n^2$$

where m and n are of the same parity

Employing Set 3 in (9), the values of a and b satisfying (1,2) are given by

$$a = 2(\alpha^3 - 3\alpha\beta^2)^2, b = 2(\beta^3 - 3\alpha^2\beta)^2$$

and using Set 4 in (9), the corresponding values of a and b satisfying (1,2) are obtained as

$$a = \frac{1}{2}(m+n)^2(m^2 + n^2)^2,$$

$$b = \frac{1}{2}(m-n)^2(m^2 + n^2)^2$$

where in the values of m and n are both even or both odd.

(II) On the system $a - b = p^3, ab = q^2$

Let a, b be two non-zero distinct positive integers such that

$$a - b = p^3, ab = q^2 \tag{10,11}$$

Elimination b between (10) and (11), one gets

$$a = \frac{1}{2} \left(p^3 + \sqrt{p^6 + 4q^2} \right) \tag{12}$$

The square root on the RHS of (12) is eliminated when

$$q = rs, p^3 = r^2 - s^2, r > s > 0 \tag{13,14}$$

and thus,

$$a = r^2, b = s^2 \tag{15}$$

It is to be noted that the values of r and s should satisfy (14). After a few calculations, it is seen that there are two sets of values to r, s as given below:

Set 3: $r = t_{3,p}, s = t_{3,p-1}, t_{3,p}$ - triangular number of rank p

Set 4:

$$r = 4k^3 + 6k^2 + 3k + 1$$

$$s = 4k^3 + 6k^2 + 3k$$

$$p = 2k + 1$$

Using set 3, the values of a, b satisfying (10,11) are given by

$$a = t_{3,p}^2, b = t_{3,p-1}^2$$

and in view of set 4, one has

$$a = (4k^3 + 6k^2 + 3k + 1)^2$$

$$b = (4k^3 + 6k^2 + 3k)^2$$

Also, the square- root on the RHS of (12) is eliminated for the following choices of p and q :

Choice (i) $p = 2ks, q = 2s^2(k^6s^2 - 1)$

Choice (ii) $p = 2\alpha\beta, q = 2(\alpha^6 - \beta^6)$

and thus, one obtains

$$a = 2s^2(k^3s + 1)^2, b = 2s^2(k^3s - 1)^2$$

and

$$a = 2(\alpha^3 + \beta^3)^2, b = 2(\alpha^3 - \beta^3)^2 \text{ respectively.}$$

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