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SOLVING HIGHER-ORDER DIFFERENTIAL EQUATIONS USING THE METHOD OF ORDER REDUCTION

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ABSTRACT
The article provides a guide to solve differential equations that may be reduced in order, and shows three types of them. The methods have been proved by examples.
KEY WORDS: *a derivative, a differential, an integral, an equation, a general solution, a private solution*

INTRODUCTION

According to Alan Turing, "Science - a differential equation" [1]. In fact, many aspects of science lead to the discovery of an unknown function that represents an event or process under consideration. Differential equations play an important role in this. Hence, differential equations – a relation between arbitrary variables and unknown functions together with their derivatives or differentials [2,420]. This article discusses the types of differential equations that may be reduced in order, and the methods of solving them.

MAIN BODY

It is known that all differential equations of second and higher orders are called as differential equations with higher order.

General view of n-order equation:

$$F(x,y,y',y'',...y^{(n-1)})=0 \quad (1)$$

Or it may be solved in relation to the higher derivative:

$$y^{(n)}=f(x,y,y',y'',...y^{(n-1)}) \quad (2)$$

Let's look at how to integrate high-order differential equations. The main method of integration is to reduce the order. In this method, the order of the variables of the given equation is changed to another equation by substituting the order of the variables that are lower than they are. However, it is not always possible to reduce the order.

It is feasible with the following types of equations:

Type 1. Equation in the form of $y^{(n)}=f(x)$

The right-hand side of this equation depends only on x. If the right-hand side of the equation depends only on x, the order of the equation is reduced by the method of integration at the same time. Consequently, we get:

$$y^{(n-1)}=\int f(x)dx+C_1$$

On this way (in order) we integrate until required times and get general solutions of the equation.

1st example.

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$y^{(n-1)} = \int f(x)dx + C_1$; initial requirements:

Solve the equation if $x=1,\ y=2,\ y^{'}=1,\ y^{''}=1$

Solution: $y^{'''} = \frac{6}{x^3}$; we find a general solution of the equation.

To do this, we integrate 3 times and get the following:

$$y^{''} = \int \frac{6}{x^3}dx = 6\int \frac{1}{x^3}dx = 6\int x^{-3}dx =$$

$$y^{''} = 6\frac{x^{-3+1}}{-3+1} \qquad y^{''} = 6\frac{x^{-2}}{-2} = -\frac{3}{x^2} + C_1$$

$$y^{''} = -\frac{3}{x^2} + C_1 \qquad y^{'} = -3\int x^{-2}dx + C_1\int dx$$

$$y^{'} = \int(-\frac{3}{x^2} + C_1)dx \qquad y' = -3\frac{x^{-1}}{-1} + C_1x + C_2$$

$$y^{'} = \frac{3}{x} + C_1x + C_2$$

$$y = \int(\frac{3}{x} + C_1x + C_2)dx$$

A general solution is $y = 3\ln x + C_1\frac{x^2}{2} + C_2x + C_3$

Now we find C_1, C_2, C_3 from the initial requirements:

$$1 = -\frac{3}{1^2} + C_1 \Rightarrow C_1 = 4$$

$$1 = \frac{3}{1} + 4\cdot 1 + C_2 \Rightarrow C_2 = -6$$

$$2 = 3\ln 1 + 4\cdot\frac{1}{2} - 6\cdot 1 + C_3 \Rightarrow C_3 = 6$$

So, a private solution will be $y = 3\ln x + 2x^2 - 6x + 6$

2nd example.

Find a general solution of the equation:

$$y^{''} = \frac{1}{1+x^2}$$

Solution.

To do this, we integrate it twice and get the followings:

$$y^{'} = \int \frac{1}{1+x^2}dx$$

Firstly, we integrate $\int \frac{1}{1+x^2}dx$

Let's integrate the 1st integral by peacemeal:

$$\int \frac{1}{1+x^2}dx = arctgx + C_1 \qquad y^{'} = arctgx + C$$

$$y = \int(arctgx + C_1)dx = \int arctgxdx + C_1\int dx$$

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$$\int arctgx dx = \begin{vmatrix} U = arctgx & dV = dx \\ dU = \frac{1}{1+x^2}dx & V = x \end{vmatrix} = xarctgx - \int x \cdot \frac{1}{1+x^2}dx =$$

$$= xarctgx - \frac{1}{2}\int \frac{d(1+x^2)}{1+x^2} = xarctgx - \frac{1}{2}\ln(1+x^2) =$$

$$= xarctgx - \ln\sqrt{1+x^2} + C_1x + C_2.$$

There will be a general solution: $y = xarctgx - \ln\sqrt{1+x^2} + C_1x + C_2.$

Type 2. The equation in the form of $y^{(n)} = f(x, y^{(k)}, y^{(k+1)}, ..., y^{(n-1)})$

is substituted like $y^{(k)} = P(x)$, $p = p(x)$ and we will get the equation with order $p^{(n-k)} = f(x, p, p', p'', ...p^{(n-k)})$.

After integrating this equation $(n-k)$ times, we define the new function that we are looking for:

$$P = \varphi(x, C_1, C_2, ...C_{n-2}),$$

$$y^{(k)} = \varphi(x, C_1, C_2, ...C_{n-k})$$

We integrate the equation k times and get a general solution.

A private form of the equation is $y'' = f(x, y')$.

Here, the order is reduced to one unit by substitution for $y' = p(x)$

3rd example.

We use $y' x\ln x = y'$ as a substitute for $y' = p(x)$

$$p' x\ln x = p \quad p' = \frac{dp}{dx} \quad \int\frac{dp}{p} = \int\frac{dx}{x\ln x}$$

$$\ln|p| = \ln\ln(x) + \ln C_1 \quad \Rightarrow \quad p = C_1\ln x$$

$$y' = C_1x$$

$$y = \int C_1x\ln x dx$$

$y = C_1x(\ln x - 1) + C_2$ - this will be a general solution.

Type 3.

An equation in the form of $y^{(n)} = f(y, y', y'', ...y^{n-1})$

As it is seen the right-hand side of this equation does not include an arbitrary variable x. We use it as a substitute for $y' = p(y)$ and the order of the equation is reduced to one unit. Here, y will be an arbitrary variable, p function that is being looked for depends on y. According to the rule of differentiation of a complex function:

$$y' = \frac{dy}{dx} = p$$

$$y'' = \frac{dy}{dx} = \frac{dp}{dy}\cdot\frac{dy}{dx} = p' \cdot p$$

$$y''' = \frac{d}{dx} = (p' \cdot p) = \frac{d}{dy}(p' \cdot p)\cdot\frac{dy}{dx} = (\frac{dp'}{dy}\cdot p + p' \cdot \frac{dp}{dy})\cdot p =$$ and so on.

$$(p'' \cdot p + p' \cdot p')\cdot p' = p'' \cdot p^2 + (p')^2 \cdot p$$

By putting $y', y'', ..., y^{(n)}$ to the equation, we get the equation of order (n-1).

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4th example.

$$y''tgy = 2(y')^2$$

We do a substitution like $y' = p(y)$ $y'' = p \cdot p'$ and get $pp'tgy = 2p^2$.

Then, we integrate $\int \frac{pdx}{p^2} = 2\int \frac{dy}{tgy}$.

$$\int \frac{dx}{p} = 2\int \frac{\cos y}{\sin y} dy \quad p\frac{dp}{dy} tgy = 2p^2 \quad \frac{pdp}{p^2} = 2\frac{dy}{tgy}$$

$$\ln p = 2\ln(\sin y) + \ln C_1$$

$$p = C_1 \sin^2 y$$

The equation in the form of $pp'tgy = 2p^2$ came to the first-order differential equation in relation to the function that its variables may be separated.

Eventually, from $y' = C_1 \sin^2 y$ we integrate $\int \frac{dy}{\sin^2 y} = C_1 \int dx$ and get a general solution.

$$-ctgy = C_1 x + C_2 \Rightarrow ctgy = C_2 - C_1 x$$

Conclusion.

General forms and methods of solving high-order differential equations have been studied. Consequently, the following types and methods of solving high-order differential equations that their order may be reduced have been revealed:

1) The right-hand side of $y^{(n)} = f(x)$ equation is integrated n times. Each integration involves one optional invariable.

2) Equation in the form of $F(x, y', y'') = 0$ or $y'' = f(x, y')$ where y has participated as a disclosed one.

3) Equation in the form of $F(y, y', y'') = 0$ or $y'' = f(x, y')$ where x has participated as a disclosed one. By the method of substitution of the equation $y' = p,\ y'' = \frac{dp}{dx}$ we get $F(x, p, \frac{dp}{dx}) = 0$.

Above-mentioned methods may serve as a guide for students to solve differential equations that their order may be reduced.

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