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**PREVENT ACCIDENT FOR CAR OVER  
DICHOTOMIZED  
MODEL IN VANET BY PREDICAMENT MESSAGES**

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**ABSTRACT**

*In a Vehicular Ad-Hoc Network (VANET), the wireless Accident Evading system problem cautionary advice to drivers before they reach a possible unsafe zone on the road. This deal an analytical model for assess the routine of plight data via wireless Accident Evading systems. we progress the prospects of a rear-end pileup of the two automobiles that travel in the same order when a precipitous incident occurs. we gauge the prospect of vehicles waning to receive the plight data. Exponential results from the model show that the number of car peal per pileup is much surpassing when a wireless Accident Evading system is not used. We also find it interesting that the number of car pileup is not directly relational to the vehicle compactness when the vehicular maneuverability traces follow the speed-quantity affiliation. By attune flow theory into VANET dissection, our model provides useful wisdom for eventual cater inventive freightage.*

**Index Terms**—*plight data, safety application, vehicular ad-hoc networks, flow theory, car pileup.*

**I. INTRODUCTION**

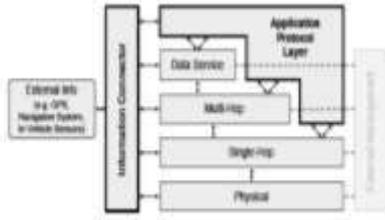
Vehicular Ad hoc network consist of mobile lumps, fixed frame and wireless interconnection to consent them to talk with each other. The most essential facility provided by these networks is driving safety. Almost 1.4 million people decease in road accidents and additional 20-50 millions are injured worldwide. Some study shows that 60% of accidents can be dodged if the driver gets the notice even before half a second of the coincidence. VANET are division of ad-hoc network waged

over vehicular domain VANET has materialized as a solution and become a key component of Intellectual Passage System. Main impartial of our model is to deliver a road safety by passing haven data.

VANET differs from other ad-hoc wireless networks of the same class in these terms:

- High processing power
- Large storage capacity
- Energy sufficiency (as work over battery of vehicle).

- Predictable movement of nodes (as vehicles are bound to follow a certain path along the road).



• VANET has vast application area classified as **Security based application**(covering Collision Avoidance, traffic analysis and interactive driving.) and **User based application**(covering entertainment domain, internet connectivity on roads and other road side services such as providing restaurant or fuel pump information).

- For easy and effective communication VANET use two prominent technologies;
- **IEEE 802.16 (Wireless MAN/WiMAX):** Wireless communication standard for MAN, designed to enable multimedia application over wireless connections ranging up to 30 miles.
- **IEEE 802.11p (WAVE):** Specially used for wireless access in vehicular domain. It enable V2V and V2I communication in the licensed ITS band of 5.9 GHz.

**VEHICULAR NETWORKS CHALLENGES**

*1) Mobility*

The basic idea from Ad Hoc Networks is that each node in the network is mobile, and can move from one place to another within the coverage area, but still the mobility is limited, in Vehicular Ad Hoc Networks nodes moving in high mobility, vehicles make connection throw their way with another vehicles that maybe never faced before, and this connection lasts for only few seconds as each vehicle goes in its direction, and these two vehicles may never meet again. So securing mobility challenge is hard problem.

**II. WIRELESS COLLISION AVOIDANCE (CA) SYSTEM**

we first demonstrate how DSRC-based wireless communication can be leveraged to improve the performance of collision avoidance applications. Next, we propose an analytical model to provide the probability of rear-end collision between two vehicles traveling in the same direction when a sudden braking situation occurs. Specifically, the proposed model accommodates features developed by traffic flow theory. The collision avoidance system is explained through a two car highway platoon example. Without loss of generality, the vehicles are traveling at a speed of 90 km/hr (25 m/s) and with an inter-vehicle spacing (headway) of 50 m. The headway denotes the distance between the front of one vehicle and the front of the subsequent vehicle. Two cases are used to illustrate braking with and without the CA system:

- In Case I, vehicle V2 does not have a CA system. The driver of V2 saw the brake light of its lead car at  $t_0 + 1.5$  s; he/she took 1.5 seconds of brake reaction time.
- In Case II, both vehicles V1 and V2 have wireless CA systems. V1 issued an emergency message at time  $t_0$ , and V2 received the message at time  $t_0 + t_l$ . The wireless latency  $t_l$  is usually less than 100 ms (0.1 seconds) in DSRC standard. As a result, when V2 received the emergency message, the driver of V2 took 1.5 s of brake reaction time and immediately initiated

There is many researches have addressed this challenge [5], [9], but still this problem unresolved.

*2) Volatility*

The connectivity among nodes can be highly ephemeral, and maybe will not happen again, vehicles travelling throw coverage area and making connection with other vehicles, these connections will be lost as each car has a high mobility, and maybe will travel in opposite direction[1],[5]. Vehicular networks lacks the relatively long life context, so personal contact of user’s device to a hot spot will require long life password and this will be impractical for securing VC.

*3) Privacy VS Authentication*

The importance of authentication in Vehicular Ad Hoc Networks is to prevent Sybil Attack that been discussed earlier [8]. To avoid this problem we can give a specific identity for every vehicle, but this solution will not be appropriate for the most of the drivers who wish to keep their information protected and private[1],[5].

*4) Privacy VS Liability*

Liability will give a good opportunity for legal investigation and this data can’t be denied (in case of accidents)[1], in other hand the privacy mustn’t be violated and each driver must have the ability to keep his personal information from others (Identity, Driving Path, Account Number for toll Collector etc.).

*5) Network Scalability*

The scale of this network in the world approximately exceeding the 750 million nodes [4], and this number is growing, another problem arise when we must know that there is no a global authority govern the standards for this network [1], [5], [7], for example: the standards for DSRC in North America is deferent from the DSRC standards in Europe, the standards for the GM Vehicles is deferent from the BMW one make a communication we have to assume that there is a limited number of cars that will receive the communication, in the future we must concentrate on getting the number higher, to get a financial benefit that will courage the commercial firms to invest in this technology [5]. an emergency deceleration (at 4 m/s<sup>2</sup>) at time  $t_0 + 1.6$  s. Finally, V2 stopped before crashing into V0 and V1.

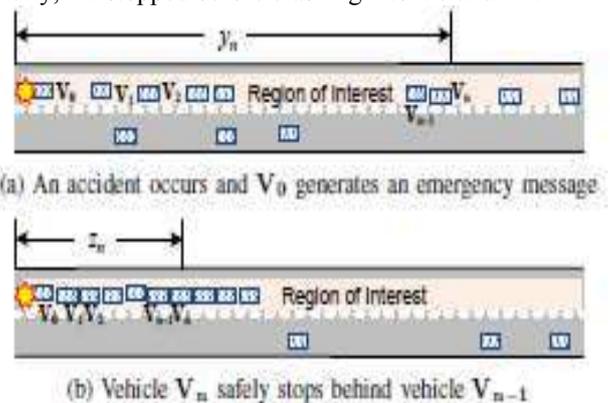


Fig. I(1): The illustration for deriving the number of crashed vehicles.

**A. Preliminary**

Specifically, we accommodate the features of the dichotomized headway model and the vehicle braking model developed by traffic flow theory. Therefore, our analytical model gives a general and comprehensive result for vehicular networking. The source vehicle V0 generates a safety message and distributes into all succeeding vehicles V1, V2, . . . , Vn and so on. To compute the chain collision probability of Vn. Let  $d_{ssd,n}^*$  be the minimum stopping sight distance

(SSD)needed for  $V_n$  without crashing into  $V_{n-1}$ . In (Fig. I(1) (a)),  $y_n$  represents the initial distance between  $V_0$  and  $V_n$  at the time  $V_n$  receives the message, the driver of  $V_n$  initiates the brake application and finally stops behind vehicle  $V_{n-1}$  (see Fig. II(1)(b)). The displacement  $z_n$  represents the maximum distance  $V_n$  needed without crashing into its preceding vehicle. We define the indicator random variable  $I_n$  by

$$I_n = \begin{cases} 1, & \text{vehicle } V_n \text{ crashes into } V_{n-1} \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$E[I_n] = \Pr[I_n = 1] = \Pr[y_n - z_n < d_{ssd,n}^*] \quad (2)$$

$$d_{ssd}^* = d_{RT} + d_{brake}^* = vt_{RT} + d_{brake}^* \quad (3)$$

$$d_{brake}^* = \frac{v^2}{2a_{d,max}} \quad (4)$$

$$d_{ssd}^* = vt_{RT} + \frac{v^2}{2a_{d,max}} \quad (5)$$

**B. Our proposed model**

we show how a vehicle crash can potentially be avoided by reducing the reaction time  $t_{RT}$ . The wireless propagation latency, typically calculated in terms of milliseconds, is significantly smaller than the cumulative drivers' reaction time, typically calculated in terms of seconds. We assume that vehicle  $V_0$  is the one that first sees the accident and is responsible for issuing the collision warning. The respective reaction time  $t(n)RT$  of vehicle  $V_n$  to brake with or without a CA system is given by

$$t_{RT}^{(n)} = \begin{cases} \sum_{i=1}^n t_{RT,i}, & \text{w/o CA} \\ \min\left(\sum_{i=1}^{n-1} t_{RT,i}, t_{l,n}\right) + t_{RT,n}, & \text{with CA} \end{cases} \quad (6)$$

In Eq. (6),  $t_{RT,i}$  denotes the brake reaction time between when

between when  $V_0$  first issues the emergency message and the driver of vehicle  $V_i$  recognizes a hazard and when she/he the wireless latency  $t_{l,n}$  denotes the time when  $V_n$  receives the message. More details will be given in the next section. Combining the vehicle braking model and from Eqs. (2) and (5), the chain collision probability of vehicle  $V_n$  can be computed by

$$\Pr[I_n = 1] = \Pr[y_n - z_n < d_{ssd,n}^*] = \Pr\left[y_n - z_n < vt_{RT} + \frac{v^2}{2a_{d,max}}\right] \quad (7)$$

As shown in Fig. 2 (b),  $z_n$  depends on the minimum safety space occupied by the preceding  $n$  vehicles. We take  $z_n = n\delta_L$ , where  $\delta_L$  is the average minimum safety space between an additional spaceheadway model in flow theory that can explain headways between bunched vehicles and free vehicles [24], [25]. The bunched vehicles closely follow preceding vehicles; the free vehicles travel without interacting with the vehicles ahead. This headway model has been proven to provide a good fit to realistic data of field observations.<sup>2</sup> Given the traffic flow rate  $q$  and the road-level traffic density  $\mu$ , the proportion  $\alpha$  of free vehicles on a road can be estimated by<sup>3</sup>

$$\alpha = \exp(-0.9q) = \exp(-0.9\mu v) \quad (8)$$

It is worth noting that the proportion of free vehicles ( $\alpha$ ) is an important parameter that affects headway distribution. Because driver behavior changes from place to place,  $\alpha$  should be calculated according to actual traffic data in a specific country, city and area [26]. It is worth noting that the proportion of free vehicles ( $\alpha$ ) is an important parameter that affects headway distribution. Because driver behavior changes from place to place,  $\alpha$  should be calculated according to actual traffic data in a specific country, city and area [26]. Another important problem in determining  $\alpha$  is classifying which vehicles are considered free vehicles and which are considered following (bunched) vehicles. (For instance, [27] defined vehicles with headways greater than 4 seconds as free vehicles.) For a bunched vehicle, its distance from the preceding vehicle is min. For a free vehicle, its distance from the preceding vehicle is min plus an exponentially-distributed distances with an average of  $1/\lambda$ . Under the above discussions, we have the following

$$g_m(s) = \frac{[\lambda(\lambda s)^{m-1}]}{(m-1)!} e^{-\lambda s} \quad (10)$$

$$G_m(s) = 1 - \left[ \sum_{k=0}^{m-1} \frac{(\lambda s)^k}{k!} \right] e^{-\lambda s}$$

Based on the consider result from equ(10), we further the minimum headway, and the proportion between bunched vehicles and free vehicles. The distribution  $F_n(\cdot)$  of  $y_n$  can be derived as

$$F_n(y_n) = (1-\alpha)^n A_n + \sum_{m=1}^n \left[ \binom{n}{m} (1-\alpha)^{n-m} \times \alpha^m G_m(y_n - n\Delta_{min}) \right] \quad (11)$$

The first term in Eq. (11) interprets the distribution conditioning on all  $V_1, \dots, V_n$  are bunched, which has probability  $I$

follows that

$$\begin{aligned}
 A_n &= \Pr[I_n = 1 | V_1, \dots, V_n \text{ are bunched}] \\
 &= \begin{cases} 1, & \text{if } n\Delta_{\min} < C(n) \\ 0, & \text{otherwise} \end{cases} \quad (12)
 \end{aligned}$$

### III. ANALYTICAL MODEL FOR VANET WITH ROADSIDE DEPLOYMENT

A. we formulate a mathematical model to investigate the effectiveness of a CA system with or without roadside deployment. There are several reasons to deploy RSU on roads. First, it improves the network connectivity. Second, it increases the message delivery options in both V2V and V2I aspects. Third, it reduces the delivery delay for sending a safety message to disconnected vehicles. In the wireless CA system, even though some further vehicles are not located in the dangerous zone, these vehicles can still utilize the immediate warning for route planning. This section evaluates the network-layer connectivity of the wireless CA system. We use the CA failure rate to indicate the probability that at least one disconnected vehicle located in the RoI failing to receive the emergency message. This output metric can be served as an important metric to determine the number of RSUs needed to be deployed in future intelligent transportation **Problem formulation**

The safety message delivery with or without roadside deployment is formulated as follows. VANET architecture containing a two-lane straight road segment with vehicles traveling in eastbound and westbound directions. The multiple-lane scenario is supported by first merging traffic on the same direction into a single stream of traffic. A source vehicle V0 crashed and it immediately generates an emergency message to its succeeding vehicles located in the region of interest S. There are n RSUs, namely U1 . . . Un. The region of interest is divided into n + 1 subsegments by RSUs U1 . . . Un. For 0 ≤ i ≤ n, let si represent the i-th subsegment. We use the notation |si| to represent the length of a road segment. We have |S| = ∑\_{i=0}^n |si|. If there is no RSU deployed on S, it follows that n = 0 and |S| = |s0|. The model derived in this section is general for two reasons: First, it can also be used to calculate collision avoidance performance when no RSU is deployed on road, i.e., n = 0. Second, for n > 0, the distance of any two consecutive RSUs need not be the same. Assuming all vehicles are equipped with DSRC devices. We make the following definitions:

**Definition 1.** A disconnected vehicle is a vehicle whose distance from the preceding vehicle in the same lane is longer than the transmission range R. For the sake of mathematical traceability, an ad hoc network can be modeled as a unit disk graph by viewing every transmitter/receiver in the broadcast network as a point in the graph and by representing the effective broadcast range of each point as a unit disk. **Definition 2.** A cluster contains a group of vehicles traveling in the same direction. The cluster's head is a disconnected vehicle. A succeeding vehicle of a cluster's tail is a disconnected vehicle. Two vehicles in the same cluster are connected, with the distance between the two consecutive vehicles being less than R.

**Definition 3.** A connected vehicle is a vehicle that belongs to a cluster of the source vehicle V0.

**Definition 4.** A CA failure situation occurs when a vehicle located in the RoI fails to immediately receive the

emergency message from the preceding vehicles. In this situation, the CA system requests an opposite vehicle to store-carry-forward the message to the destination.

#### B. Deriving the CA failure rate:

Excluding the source vehicle V0, there are m vehicles, denoted by V1 . . . Vm, are located in a region of interest S. By convention, assume V0 = U0 and Vm = Un+1. Let the connectivity index C(Ui, Ui+1) denote the connectivity probability for vehicles located in the ith road subsegment with the RSUs, Ui and Ui+1, at both ends. Then, the connectivity probability for vehicles located in the road segment S can be represented by C(U0, U1)C(U1, U2) . . . C(Un, Un+1). That is, a fully connected VANET is formed in all subsections individually. The existence of a CA failure situation in the region of interest becomes

$$P^* = 1 - \prod_{i=0}^n C(U_i, U_{i+1})$$

The derivation of C(Ui, Ui+1) is described as follows: Consider a deployment rate of κ roadside units per meter. Denote θ(s) as the probability that all vehicles located in an s-meter road section are fully connected with DSRC/relay devices at both ends. We have C(Ui, Ui+1) = θ(|si|) and

$$P^* = 1 - \prod_{i=0}^n C(U_i, U_{i+1}) = 1 - \prod_{i=0}^n \theta(|s_i|)$$

the CA failure rate P\* derived in previous eq.,

$$\begin{aligned}
 P^* &= 1 - \prod_{i=0}^n \theta(|s_i|) \\
 &= 1 - \prod_{i=0}^n e^{-\mu^* |s_i|} \\
 &\quad \times \sum_{k=0}^{\lfloor |s_i|/R \rfloor} \left\{ \frac{(-1)^k}{k!} [\mu^* (|s_i| - kR)]^{k-1} \right. \\
 &\quad \left. \times [k + \mu^* (|s_i| - kR)] e^{\mu^* (|s_i| - kR)} \right\}
 \end{aligned}$$

### IV. RESULTS AND DISCUSSION

For evaluation purposes, we outline the VANET topology, mobility model, and data traffic model in the customized event-driven Monte-Carlo C++ simulator. In each simulation round, we use the dichotomized headway model to generate the vehicles' locations. We consider the maximum vehicle deceleration rate of 4 m/s<sup>2</sup> and the safety headway Δmin of 1.5v + δL, where the average driver's reaction time of 1.5 seconds. The vehicles' locations are then used to simulate the distance to the accident site. The source vehicle V0 (i.e., the vehicle nearby the accident site) is responsible for issuing the emergency message via the CA system. The average latency tlf for a transmission attempt is set to be 20 ms with parameters Gt = Gr = 4 dB, Pt = 2 dBm, and PrxTh = -90 dBm.

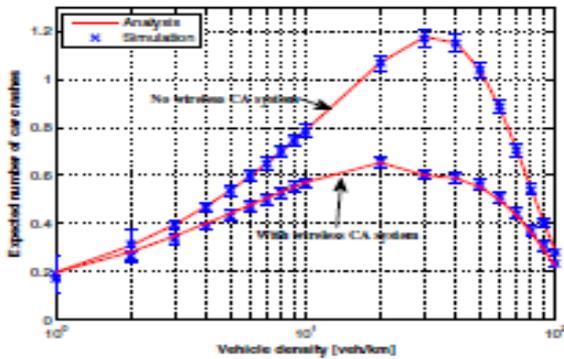


Fig IV(1):The expected density of car collision against traffic density.

the expected number of car crashes can be computed by,

$$\begin{aligned}
 E[n_c] &= E\left[\sum_{n=1}^{\infty} I_n\right] = \sum_{n=1}^{\infty} E[I_n] \\
 &= \sum_{n=1}^{\infty} \Pr[I_n = 1]
 \end{aligned}$$

We make the following observations:

- 1) Without a wireless CA system, the damage after an accident would be more serious compared to that with a wireless CA system.
- 2) With a wireless CA system, a car collision can only be slightly improved by shortening the wireless latency, which is calculated in terms of milliseconds. The numerical result showed that utilizing a wireless CA system in road-level densities of 10 veh/km and 50 veh/km reduces the number of car crashes by 27% and 47%, respectively.
- 3) One of our interesting observations is that the number of car crashes in an accident is not a monotonically increasing function of vehicle density when we consider Greenberg’s logarithmic model developed by flow theory. In realistic traffic flow, when traffic density is high, vehicles need to move slowly because they are blocked by preceding vehicles.

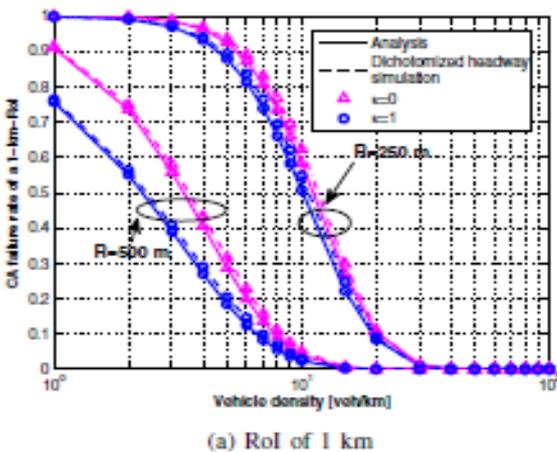


Fig IV(2):The CA failure rate in an ROI, where Δ<sub>min</sub>=10m.

TABLE I  
SIMULATION PARAMETER USED IN NS-2.

Parameter	Value
Speed Range	(80, 100) [km/hr]
Traffic Flow Rate	864, 1728 [veh/hr]
Region of Interest	5 km
Transmission Range	250, 500 [m]
RSU deployment rate	0-4 RSU/km
Message Payload	214 bytes
Carrier Frequency	5.9 GHz
Channel Bandwidth	10 MHz

### V. CONCLUSIONS

One promising aspect of VANET is that it can considerably improve road safety and travel comforts by enabling inter vehicle communications. Among a vast array of potential applications, emergency messaging has attracted much attention in the literature. When vehicles are connected to VANET, drivers can immediately receive emergency messages via direct transmission. In such instances, drivers have enough time to react to hazardous situations appropriately. For instance, vehicles near the accident site can slow down or stop before colliding with the preceding vehicle, while vehicles further away can quickly change their lanes or make detour/reroute decisions accordingly. This paper developed mathematical models to analyze the performance of a wireless collision avoidance system with or without employing RSUs as ad-hoc relays. The efficacy of the proposed framework has been analyzed and validated by extensive simulation results. The numerical results indicated that the vehicle density in a critical range is more prone to chain collisions. The CA system must keep broadcasting the latest road information more frequently to drivers when the detection of traffic density by the VANET safety application is within this critical range. As a final remark, the traveling speed and the deceleration rate vary depending on traffic regulations, vehicle type, driver behavior and country. The requirements and constraints for collision avoidance are based on engineering insights and practical limitations. This was a first attempt to formulate the performance of DSRC safety applications by integrating network and flow theory so that the analysis is more comprehensive and realistic for transportation planning.

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