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UNSTEADY BOUNDARY LAYER FLOW OF A NANOFLUID OVER A STRETCHING/SHRINKING SHEET WITH A CONVECTIVE BOUNDARY CONDITION

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ABSTRACT

An unsteady boundary layer flow of a nanofluid past a stretching/shrinking sheet with a convective boundary condition is studied. The effects of the unsteadiness parameter, stretching/shrinking parameter, convective parameter, Brownian motion parameter and thermophoresis parameter on the local Nusselt number are investigated. Numerical solutions to the governing equations are obtained using a shooting method. The results for the local Nusselt number are presented for different values of the governing parameters. The local Nusselt number decreases as the stretching/shrinking parameter increases. The local Nusselt number is consistently higher for higher values of the convective parameter but lower for higher values of the unsteadiness parameter, Brownian motion parameter and thermophoresis parameter.

KEYWORDS: *Unsteady boundary layer; Stretching/shrinking sheet; Heat transfer; Nanofluid*

INTRODUCTION

The expression "nanofluid" which was first utilized by Choi and East-man [1] alludes to the scatterings of nanometer-sized particles in a base fluid, for example, water, ethylene glycol and propylene glycol, to expand their heat conductivities. Nanofluids have pulled in much consideration as another age of coolants for different modern and car applications. Subsequently, numerous papers on nanofluids have been distributed, for example, the papers by Xuan and Li [2], Xuan and Roetzel [3], Eastman et al. [4], Tiwari and Das [5] and Buongiorno [6]. In his paper, Buon-giorno [6] built up an expository model for convective transport in nanofluids which considers the Brownian dissemination and thermophoresis impacts. Buongiorno demonstrate was utilized in numerous ongoing papers, e.g. Neild and Kuznetsov [7– 9] and Bachok et al. [10, 11] among others. The boundary layer flow over a stretching sheet is imperative in applications, for example, expulsion, wire drawing, metal turning, hot rolling, and so forth [12]. The flow over a stretching sheet was first concentrated by Crane [13] who introduced an exact analytical solution for the unfaltering two-dimensional

Nomenclature

A	stretching parameter	u, v	velocity component
C	concentration	U_w	stretching velocity
C_{fx}	skin friction coefficient	v_0	mass flux velocity
c_p	specific heat	x, y	direction component
C_w	concentration at the wall		
C_∞	ambient concentration	**Greek symbols**	
D_B	Brownian diffusion coefficient		
D_T	thermophoresis diffusion coefficient	η	dimensionless similarity variable
f	dimensionless flow function	μ	dynamic viscosity of the fluid
k	thermal conductivity	υ	kinematic viscosity of the fluid
L_e	Lewis number	ρ_f	density of the fluid
N_b	Brownian motion parameter	$(\rho c)_f$	heat capacity of the fluid
N_t	thermophoresis parameter	$(\rho c)_p$	heat capacity of a nanoparticle
N_{ux}	local Nusselt number	ψ	stream function
P	fluid pressure	θ	dimensionless temperature
$\mathrm{P_r}$	Prandtl number	ϕ	dimensionless concentration
q_m	wall mass flux	τ	relative heat capacity of the fluid
q_w	wall heat flux	τ_w	surface shear stress
Re_x	local Reynolds number	γ	Biot number
S	mass flux parameter	α	thermal diffusivity
S_{hx}	local Sherwood number	**Subscripts**	
T	temperature of the fluid		
T_W	constant temperature at the wall	∞	condition at the free flow
T_∞	ambient temperature	w	condition at the wall/surface

flow over a stretching plate in a calm fluid. Be that as it may, as of late, the examination on the flow over a shrinking sheet has accumulated significant consideration. Miklav ˇci ˇc and Wang [14] started the investigation of flow over a shrinking sheet. They found that the vorticity isn't
restricted to a boundary layer and an enduring flow can't exist without applying promotion compare suction at the boundary. From that point onward, various examinations rise, exploring distinctive parts of this issue, for example, those concentrated by Wang [15], Fang [16] and Zaimi and Ishak [17], to give some examples. In the boundary layer flow and heat transfer examination, it is standard for the flow to be accepted as relentless. All things considered, in many building applications, flimsiness turns into a vital piece of the issue where the flow moves toward becoming time subordinate [11, 18, 19]. Hence, inspired by this, we broaden the investigation of Bachok et al. [11] to the instance of convective surface boundary condition. For quite a while, consistent surface temperature and heat flux are generally utilized. In any case, there are times when heat transfer at the surface depends at first glance temperature, as what for the most part happens in heat transfers. In this circumstance, convective boundary condition is utilized to supplant the state of endorsed surface temperature. Aziz [20] utilized the convective boundary condition in his examination to consider the heat transfer attributes for the Blasius flow. Ishak [21] presented the impacts of suction and infusion at the boundary. Makinde and Aziz [22] examined the boundary layer flow of a nanofluid past a stretching sheet with a convective surface boundary condition. Pattnaik et al. [23-30] investigated the study of MHD fluid flow in different papers and also they considered some investigation on nanofluid flow. The dependency of the local Nusselt number and local Sherwood number on six parameters, to be specific the stretching/shrinking, unsteadiness, convective, Brownian motion and thermophoresis parameter and Lewis number is the main focus of the present analysis.

Mathematical formulation:

Consider an unsteady, two-dimensional (x, y) boundary layer flow of a viscous and incompressible fluid over a stretching/shrinking sheet immersed in a nanofluid. It is assumed that at time $t = 0$, the velocity of the sheet is $U_w(x,t) = 0$. The un-steadiness in the flow field is caused by the time-dependent velocity of the stretching sheet, which is given by $U_w = Ax/t$ where $A > 0, t > 0$ [11, 31–33]. It is also assumed that the constant mass flux velocity is $v_0(x,t)$ with $v_0(x,t) < 0$ for suction and $v_0(x,t) > 0$ for injection or withdrawal of the fluid. The nanofluid is confined to $y > 0$, where y is the coordinate measured normal to the stretching/shrinking surface as shown in Fig. 1. It is further assumed that the bottom surface of the sheet is heated by convection from a hot fluid at temperature T_f which provides a heat transfer coefficient h. The surface temperature T_w is the result of a convective heating process characterized by the hot fluid.

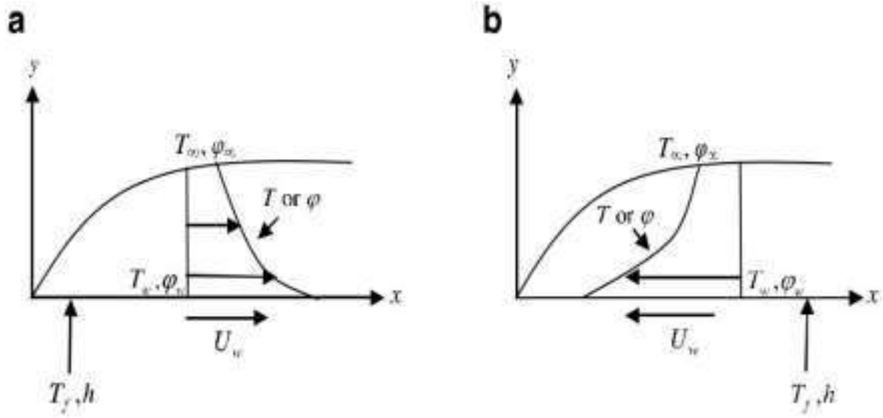


Fig. 1 Geometry of the problem for (a) stretching (b) shrinking sheets

The governing equations for the problem can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{-1}{\rho_f}\frac{\partial P}{\partial x} + \upsilon\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{-1}{\rho_f}\frac{\partial P}{\partial y} + \upsilon\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \tau\left[D_B\left(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x} + \frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right) + \left(\frac{D_T}{T_\infty}\right)\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right]\right] \tag{4}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{5}$$

with boundary conditions:

$$t = 0: u(x,y,t) = v(x,y,t) = 0, T(x,y,t) = T_w, C(x,y,t) = C_w$$

$$t > 0: \begin{cases} u(x,t) = \sigma U_w(x,t), v(x,t) = v_0(x,t) \\ -k\dfrac{\partial T}{\partial y} = h(T_f - T_w), C(x,t) = C_w \end{cases} \quad \text{at} \quad y = 0 \tag{6}$$

$$u(x,y,t) \to 0, v(x,y,t) \to 0, T(x,y,t) \to T_\infty, C(x,y,t) \to C_\infty \quad \text{as } y \to \infty \tag{7}$$

With the help of stream function and the following similarity transformation:

$$u=\frac{\partial\psi}{\partial y},v=-\frac{\partial\psi}{\partial x},\eta=y/\sqrt{\upsilon t},\psi=Ax\sqrt{\frac{\upsilon}{t}}\,f(\eta),\theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty},\phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \quad (8)$$

Eqs. (2-5) are now reduced to:

$$f'''+A\left(ff''-f'^2\right)+f'+\frac{\eta}{2}f''=0 \quad (9)$$

$$\frac{1}{P_r}\theta''+\left(Af+\frac{\eta}{2}\right)\theta'+N_b\theta'\phi'+N_t\left(\theta'\right)^2=0 \quad (10)$$

$$\phi''+\mathrm{P_r}\,L_e\left(Af+\frac{\eta}{2}\right)\phi'+\frac{N_t}{N_b}\theta''=0 \quad (11)$$

and the boundary conditions are reduced as:

$$\begin{gathered}f(0)=S,\ f'(0)=\sigma,\theta'(0)=-\gamma[1-\theta(0)],\phi(0)=1\\ f'\to 0,\quad \theta\to 0,\quad \phi\to 0\quad \text{as}\quad \eta\to\infty\end{gathered} \quad (12)$$

where

$$\begin{gathered}\mathrm{P_r}=\frac{\upsilon}{\alpha},L_e=\frac{\alpha}{D_B},N_t=\frac{\tau D_T(T_f-T_\infty)}{\upsilon T_\infty},h=\frac{c}{\sqrt{t}}\\ N_b=\frac{\tau D_B(C_w-C_\infty)}{\upsilon},S=-\frac{v_0(x,t)}{A\sqrt{\upsilon/t}},\gamma=\frac{c}{k}\sqrt{\upsilon}\end{gathered} \quad (13)$$

Physical quantities:

The physical quantities of engineering interest are the Skin friction coefficient C_{fx}, local Nusselt number Nu_x and local Sherwood number Sh_x which are defined as:

$$C_{fx}=\frac{\tau_w}{\rho_f U_w^2},Nu_x=\frac{xq_w}{k(T_f-T_\infty)},Sh_x=\frac{xq_m}{D_B(C_w-C_\infty)} \quad (14)$$

The wall shear stress and heat transfer from the plate, respectively, are given by,

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0},q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0},q_m=-D_B\left(\frac{\partial C}{\partial y}\right)_{y=0} \quad (15)$$

So from equation (14) we get, Skin friction coefficient, Nusselt number and Sherwood number respectively are defined as:

$$C_{fx}\sqrt{\mathrm{Re}_x}=A^{-1/2}f''(0),Nu_x/\sqrt{\mathrm{Re}_x}=-A^{-1/2}\theta'(0),Sh_x/\sqrt{\mathrm{Re}_x}=-A^{-1/2}\phi'(0) \quad (16)$$

where $\mathrm{Re}_x=\frac{xU_w(x)}{\upsilon}$.

RESULTS AND DISCUSSIONS

The set of ordinary differential Eqs. (9)– (11) with the boundary conditions (12) were solved numerically using a shooting method. In this analysis, all profiles satisfy the far field boundary conditions (12) asymptotically but with different boundary layer thicknesses. The asymptotic boundary conditions (12) at $(\eta = \infty)$ are replaced by $(\eta = 6)$ as customary in the boundary layer analysis. This choice is adequate for the velocity, temperature and concentration profiles to reach the far field boundary conditions asymptotically. This problem of a regular (viscous) fluid involves the parameters: Prandtl number $(\mathrm{P_r})$, stretching/shrinking parameter (A), suction/injection parameter (S), unsteadiness parameter (β), convective parameter/Biot number (γ), Brownian motion (N_b), thermophoresis parameters (N_t) and Lewis number (L_e). Variation of velocity profile for different pertinent parameters is discussed in Fig.2 (a-c). Fig. 2(a) shows that increasing values of A decreases the velocity profile but after $\eta = 5$, it goes asymptotically. But in Fig. 2(b), for increasing values of stretching/shrinking velocity parameter (σ), velocity profile increases but for $\eta > 1$ it decreases asymptotically. Fig. 2(c) shows that increasing values of S decreases the velocity asymptotically. Variation of Temperature profile for different pertinent parameters is discussed in Fig. 3(a-c) and 4(a-c). It is interesting to note that temperature profile gets decelerated for increasing values of all the parameters i.e., A, $\mathrm{P_r}$ and S which is evident in Fig. 3. Fig. 4(a) and (b) show the increasing behaviour of temperature profile for increasing values of Brownian motion (N_b) and thermophoresis parameters (N_t) but reverse trend has been occurred for increasing values of convective parameter, Biot number (γ) as in case of 4(c). Figs. 5 and 6 show the variation of concentration profile for increasing values of stretching/shrinking parameter (A), Prandtl number $(\mathrm{P_r})$, suction/injection parameter (S), Brownian motion (N_b), thermophoresis parameters (N_t) and Lewis number (L_e). Concentration profile decreases for increasing values of $A, \mathrm{P_r}, S, N_b$ and L_e but it increases for an increasing values of N_t. Skin friction coefficient decreases for increasing values of all the parameters A, σ and S which can be observed in Fig. 7(a-c). Fig. 8 is the evidence of variation of Nusselt number which confirms the increasing behaviour with increasing values of N_b and N_t but for increasing values of $\mathrm{P_r}$, it increases near the boundary and then after it decreases asymptotically. In Fig. 9, Sherwood number increases near the boundary for both N_b and L_e then after it decreases asymptotically but reverse trend has been observed for increasing values of N_t.

CONCLUSIONS

The unsteady boundary layer flow of a nanofluid past a stretching/shrinking sheet with a convective boundary condition was studied. The effects of stretching/shrinking parameter, convective parameter, Brownian motion parameter and thermophoresis parameter on Skin friction coefficient, local Nusselt number and local Sherwood number were studied. Numerical solutions to the governing equations were obtained using a shooting method. The results are presented for different values of the governing parameters. Skin friction coefficient decreases for all the parameters A, σ and S .The local Nusselt number increases as the Brownian motion parameter, thermophoresis parameter and Prandtl number increase. Near the boundary, local Sherwood number increases for both Brownian motion parameter and Lewis number but reverse effect is observed for and thermophoresis parameter.

Fig.2 Variation of Velocity Profile $f'(\eta)$

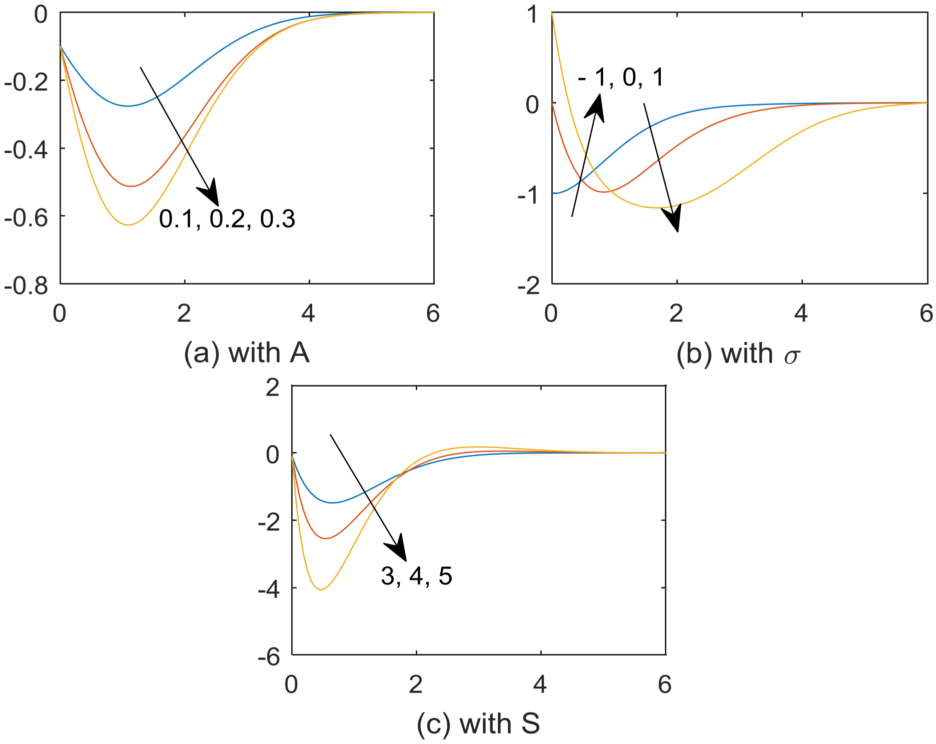


Fig.3 Variation of Temperature Profile $\theta(\eta)$

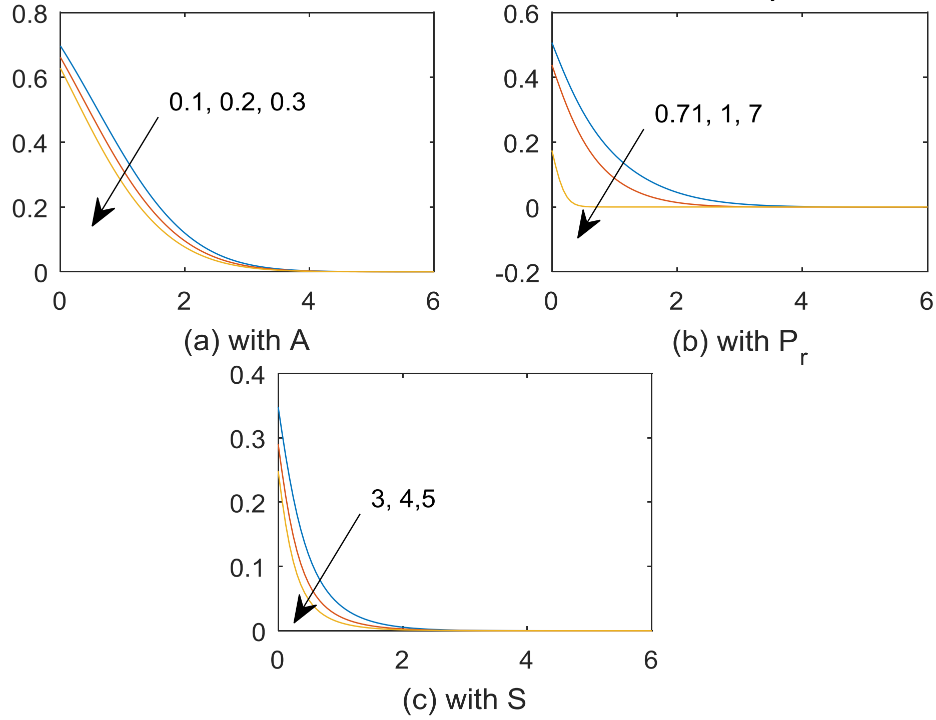


Fig.4 Variation of Temperature Profile $\theta(\eta)$

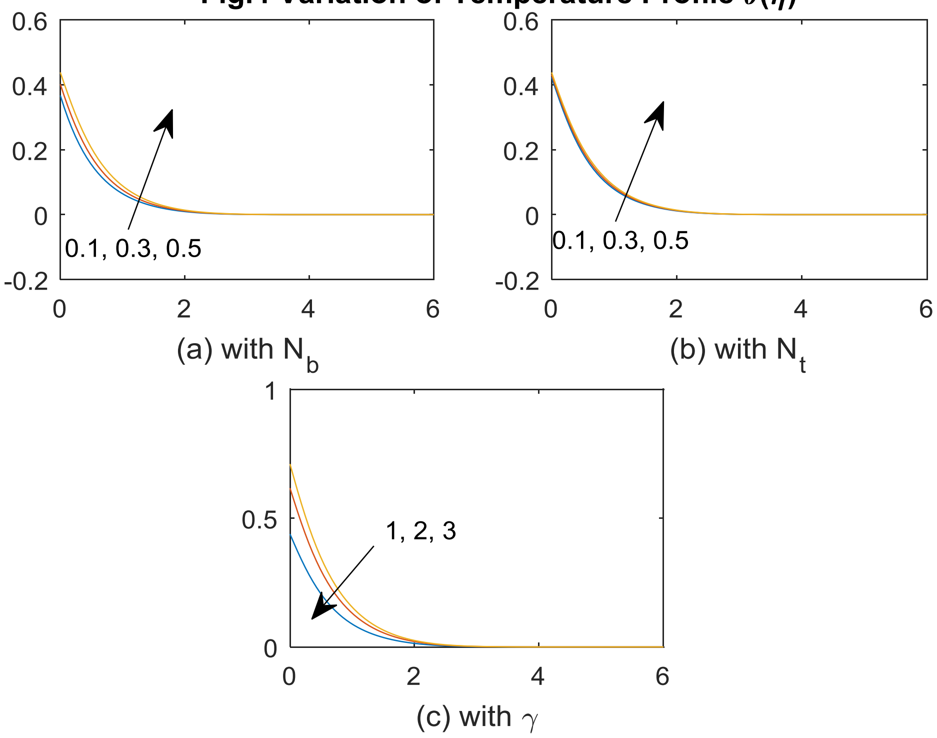


Fig.5 Variation of Concentration Profile $\phi(\eta)$

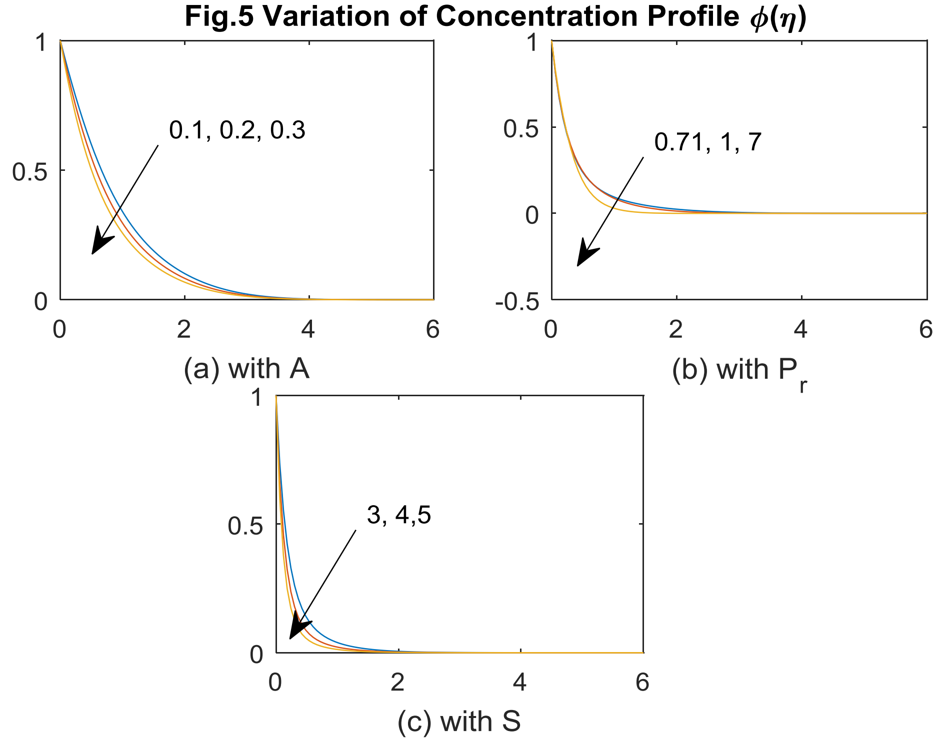


Fig.6 Variation of Concentration Profile $\phi(\eta)$

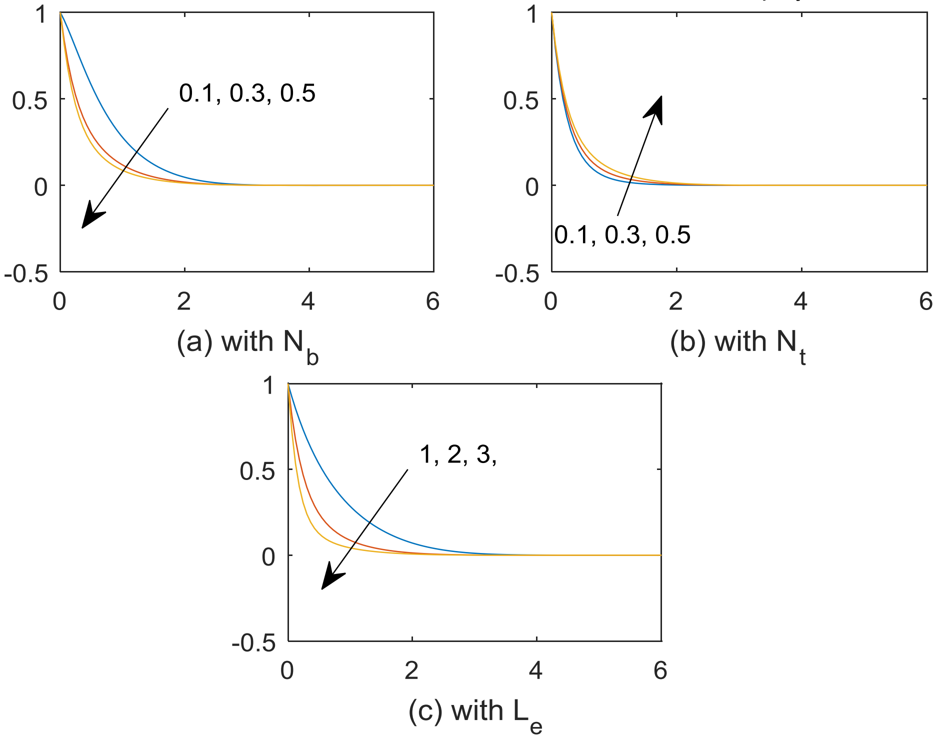


Fig.7 Variation of Skin Friction Coefficient C_{fx}

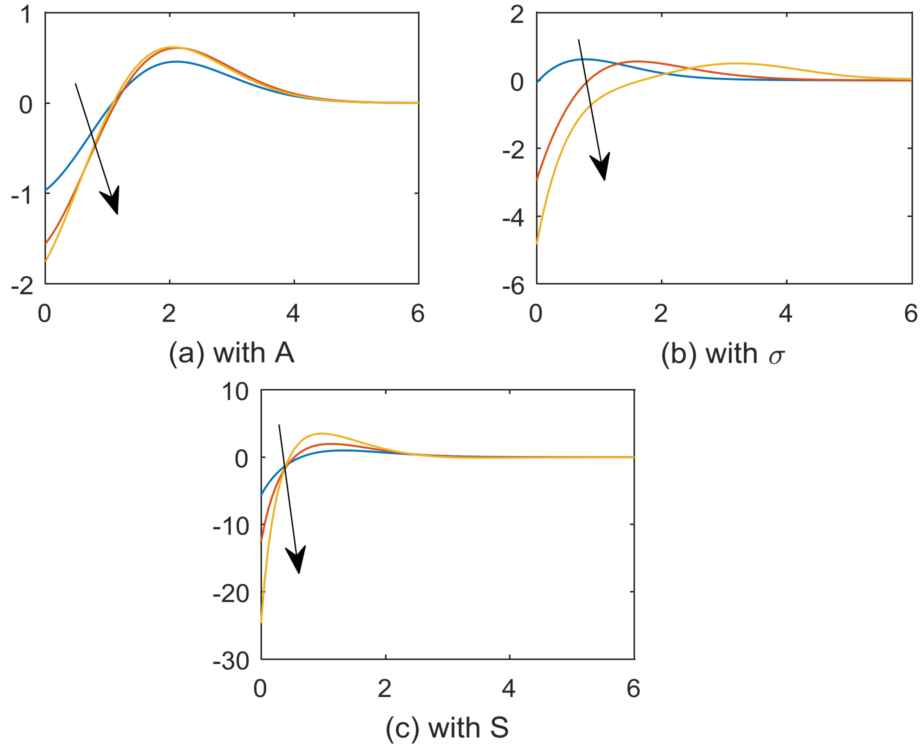


Fig.8 Variation of Nusselt Number Nu_x

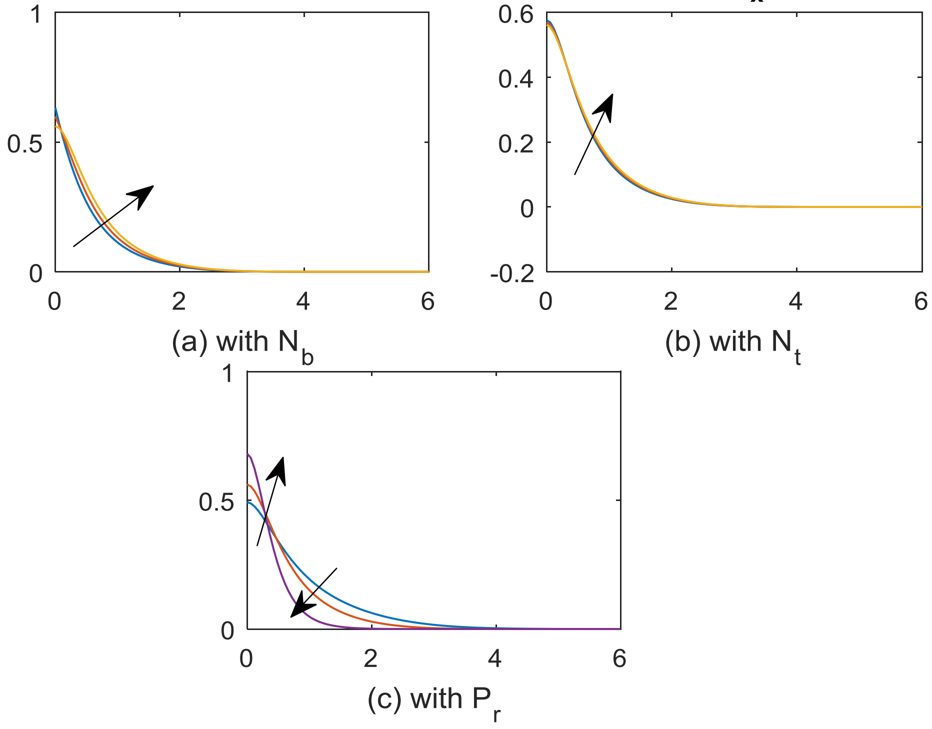
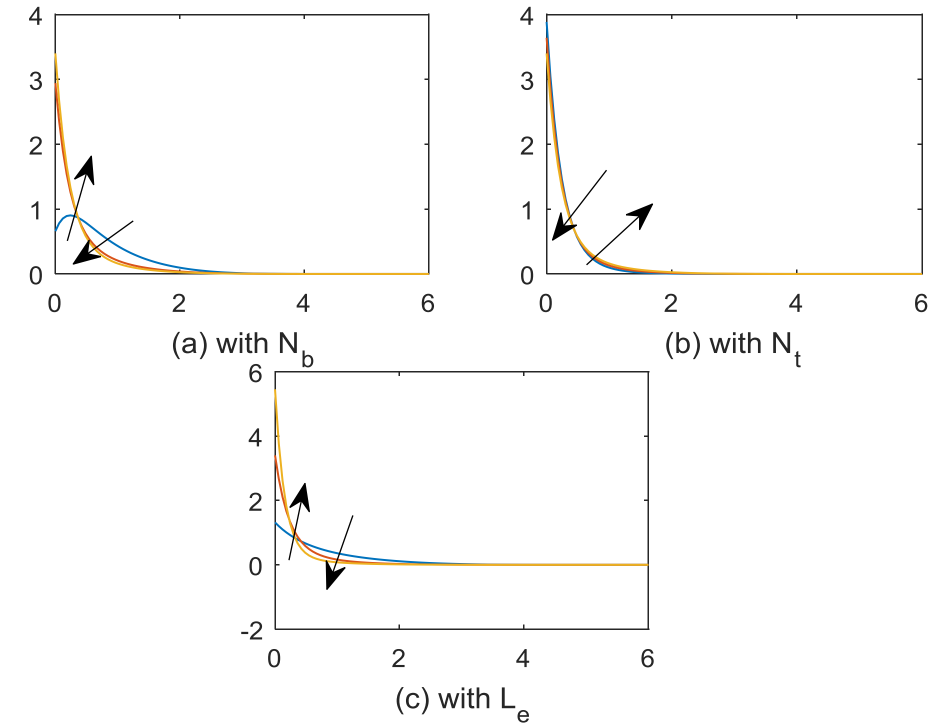


Fig.9 Variation of Sherwood Number Sh_x



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