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**INFLUENCE OF STRUCTURAL INERTIA: A  
NUMERICAL STUDY ON SIMPLY SUPPORTED BEAM**

**K Priyanka<sup>1</sup>**

<sup>1</sup>M. Tech Student at Jawaharlal Nehru  
Technological University,  
Anantapur, Andhra Pradesh,  
India

**Vaishali. G. Ghorpade<sup>2</sup>**

<sup>2</sup>Professor,  
Dept. of Civil Engineering,  
JNTU College, Anantapur,  
Andhra Pradesh,  
India

**H. Sudarsana Rao<sup>3</sup>**

<sup>3</sup>Professor and Director Academic and  
Audit,  
JNTU College, Anantapur,  
Andhra Pradesh,  
India.

**ABSTRACT**

*Dynamic loading consideration became one of the important criteria while designing any industrial, commercial or military structures, because these structures often subjected to blast, impact or seismic load etc. Moreover, these are the massive structures; inertia is one of the factors need to be considered among the all resisting forces like structural damping and stiffness. The isotropic damage model is the simple model generally used for damage modelling. But its mesh dependency nature became the constraint. So here in this paper gradient based non-local damage model was developed in addition to that this model was enhanced with structural inertia term. User modifiable damage parameters can be tuned to match with experimental results. Current paper deals with a reinforced concrete slab of dimensions 1300mm\*1100mm\*120mm with 25 mm cover are simply supported on its sides. The reinforced concrete specimen was subjected to TNT explosion with an explosive charge of 2.09kg. The standoff distance for the TNT explosion location used is 600mm. The numerical analysis involves Fluid-structure interaction, where explosion simulated with the fluid pressure that is allowed to interact with the structure. One way coupling was done to minimize the time for simulation.*

**KEYWORDS:** *Isotropic damage models, damage modelling, structural inertia, mesh dependency, non-local damage model and fluid structure interaction.*

## 1. INTRODUCTION

Concrete is widely used in the construction of civil structures, dams, nuclear reactor containment and various defence structures. These structures are subjected to extreme loading conditions like blast and impact loading causing large strains, high strain rates, spalling, fracture and erosion. It, therefore, becomes important to take the structural response due to this type of loading into consideration during the design stage. Structures must be built to resist this type of loading to the extent possible. Even where it is difficult to prevent damage, the designer should be able to predict appropriate damage that may be caused by different kinds of loading.

A few experimental studies on the response of reinforced concrete slab were reported such as According to A.G. Razaqpur et.al[1] (2007) worked on 'Blast loading response of reinforced concrete panels reinforced with externally bonded GFRP laminates. G. Thiagarajan, et.al[2] (2015) presented 'Experimental and finite element analysis of doubly reinforced concrete slabs subjected to blast loads'. G.C. Mays et.al[3] (1999) reported 'Response to blast loading of concrete wall panels with openings'. C.F. Zhao et.al[4] (2013) presented 'Damage Mechanism and mode of square reinforced concrete slab subjected to blast loading'. X.Q. Zhou et.al[5] (2008) reported 'Numerical prediction of concrete slab response to blast loading'. Few studies on blast load and strain rate in concrete formulations presented such as E.L. Lee et.al[6] (1968) et.al studied on Adiabatic expansion of high explosive detonation products. L.J.Malvar et.al[7] (1998) on 'Review of strain rate effects for concrete in tension'. Peerlings et.al[8] (1996) submitted their work on 'Gradient enhanced damage for quasi-brittle materials, W. Sun[9], 2009, 'Experimental studies on reinforced concrete (RC) slabs subjected to blast loads'.

An efficient numerical multi-physics code FLUIDYN-MP is used as the means of investigation. FLUIDYN-MP is a general purpose multi-physics code that employs a Finite Element method. The focus of the paper is on damage modelling of the simply supported slab under blast loading. Damage implementation used is based on the gradient formulation of the damage model is derived from the non-local theory according to R.H.J Peerlings et.al [8]

## 2. MATERIAL FORMULATION

In the current dissertation rely on the constitutive material model for concrete under high strain rates by U.Haeussler Combe et.al [11 and 12] is employed. According to that, the formulation is based on strain and gradient part to capture the non-local damage also. The fundamental assumption of this model is that they considered the damage activation is not instantaneous but the damage is retarded by inertia, which will arise from microcracking. If higher loading rates subjected to the structural component damage activation will be much more delayed because the inertial effects are more pronounced and resulting in higher strength. The simple isotropic damage model captures only local damage and constrained by mesh dependency. So gradient based non-local damage model in addition to that the inertial effect was preferred.

## 3. CONSTITUTIVE MODEL:

An existing isotropic damage model is used to model concrete; a rate independent model for concrete is described initially for both static and dynamic analysis. The details of the theory, implementation and some initial results are presented below. One of the methodologies to describe concrete behavior is by using damage mechanics. Initially, a local damage model is described, followed by a non-local approach to circumvent the problems arising by using a local model.

### 3.1 Local Damage Model

The following section describes the local damage model, presents a general formulation and the implementation aspects. Continuum damage mechanics is a theory which describes the progressive loss of the integrity due to propagation and coalescence of micro cracks and micro-defects. These changes in the microstructure lead to a degradation of material stiffness observed at the macro scale. The simplest version is the isotropic damage model where the behavior of the micro cracks is independent of orientates on and depends on only one scalar damage variable  $\omega$ . Therefore, the degradations of the elastic moduli in different directions are proportional and independent of the loading direction. In the simplest isotropic elasticity based damage model the constitutive relation reads:

$$\sigma = (1 - \omega)D:\varepsilon \tag{1}$$

Where D represents the tensor of elastic moduli and  $\omega$  is the variable of damage which is in between 0 (initially undamaged state) and 1 (fully damaged state). The growth of damage is governed by the damage evolution law which is generally written as

$$\omega = f(\kappa) \tag{2}$$

Where  $\kappa$  can be called as a scalar measure of the largest strain level reached in the history of the material. Therefore  $\kappa$  can be denoted as function of the strain tensor which can be written as:

$$\kappa(\varepsilon_{eq}) = \max(\varepsilon_{eq}, \kappa_{max}) \tag{3}$$

$\varepsilon_{eq}$  will be used is as the equivalent strain

To compute the value of equivalent strain various methods can be adopted. In the concrete point of view, damage accumulates in tension only, and may be highlighted and provided as given below:

$$\varepsilon_{eq} = \sqrt{\sum_{i=1}^3 \langle \varepsilon_i \rangle^2} \tag{4}$$

$\langle \cdot \rangle$  is the positive operator when applied to a scalar x gives  $\langle x \rangle = \frac{1}{2}(x + |x|)$  and  $\varepsilon_i$  are the principal strains.

Another replacement definition for equivalent strain is given by remodel von mises criterion as

$$\varepsilon_{eq} = \frac{\eta - 1}{2\eta(1 - 2\nu)} I_1 + \left(\frac{1}{2\eta}\right) \sqrt{\frac{1}{2\eta} \frac{(\eta - 1)^2}{(1 - 2\nu)} I_1^2 + \frac{12\eta}{(1 + \nu)^2} J_2} \tag{5}$$

The damage growth possibility probably concluded with the help of a loading function given by equation (5), which is a in terms of equivalent strain:

$$f(\varepsilon_{eq}) = \varepsilon_{eq} - \kappa(\varepsilon_{eq}) \tag{6}$$

The loading function follows the Kuhn –Tucker relations:

$$f \dot{\kappa} = 0 \dots f \leq 0 \dots \dot{\kappa} \tag{7}$$

Several damage evolution laws exist and in this proposal an exponential softening damage law would be used, Where gradual convergence for infinite strain values can be reached.

$$\omega(\kappa) = 1 - \frac{\kappa_i}{\kappa} [1 - \alpha + \alpha \exp(-\beta(\kappa - \kappa_i))] \tag{8}$$

where  $\kappa_i$  is the damage threshold. There will be damage value null if  $\kappa < \kappa_i$ . The terms  $\alpha$  and  $\beta$  named model parameters can easily influence the residual stress level and the steepness of the softening curve respectively. The alternative damage evolution law defined as follows

$$\omega(\kappa) = 1 - \frac{\kappa_i}{\kappa} \exp\left(-\frac{\kappa - \kappa_i}{\kappa_c - \kappa_i}\right) \tag{9}$$

Another damage law is given as

$$D(\kappa) = \begin{cases} 0 & \kappa < e_0 \\ 1 - e^{-\frac{(\kappa - e_0)^{g_d}}{e_d}} & \kappa > e_0 \end{cases} \quad (10)$$

Since concrete behavior is non-linear, linearization should be done to get the solution in an incremental form; therefore a tangent stiffness matrix is part of the solution procedure which is described below. The tangent stiffness matrix is required for implicit analysis.

In order to model rate dependency, the standard non-local gradient damage model equation is enhanced by an inertia term as given in eqn.[13]. Details on the gradient damage model and its finite element implementation can be found in Hausslers Combe et.al 2011 [13] and therefore only discussed briefly in the next section. The primary idea behind the following inertia enhanced equation is, that cracks cannot propagate arbitrarily fast as it involves the movement of masses at a micro-scale level and hence the material stiffness is affected by the increment in damage only after a certain time delay.

$$m_\kappa \ddot{\bar{\kappa}}(x) + \bar{\kappa}(x) - c\Delta\bar{\kappa}(x) = \kappa(x) \quad (11)$$

with  $m_\kappa$  the damage inertia leading to a delay in damage evolution,  $\bar{\kappa}$  the non-local equivalent strain,  $\kappa$  the local equivalent strain,  $c$  the non-local radius,  $\Delta$  the Laplacian operator. After solving for  $\ddot{\bar{\kappa}}$  it is replaced by  $\kappa$  in eqn.3 to evaluate the damage variable  $D$ .

### 3.2 Numerical Discretisation

A brief description of discretisation is presented below, for a complete procedure the viewer can preferred to [13]. To begin with, for a finite element implementation the weak form of eqn.15 leads to

$$\int_V \delta\bar{\kappa} [\kappa - m_\kappa \ddot{\bar{\kappa}} - \ddot{\bar{\kappa}} + c\Delta\bar{\kappa}] dV = \int_V \delta\bar{\kappa} \kappa dV - \int_V m_\kappa \delta\bar{\kappa} \ddot{\bar{\kappa}} dV - \int_V \delta\bar{\kappa} \bar{\kappa} dV + \int_V c\delta\bar{\kappa} \Delta\bar{\kappa} dV = 0 \quad (12)$$

Using the product rule and Gauss theorem eqn.12 reduces to

$$\int_V m_\kappa \delta\bar{\kappa} \ddot{\bar{\kappa}} dV + \int_V \delta\bar{\kappa} \bar{\kappa} dV + \int_V c\nabla\delta\bar{\kappa} \cdot \nabla\bar{\kappa} dV = \int_V \delta\bar{\kappa} \kappa dV + \int_A c\delta\bar{\kappa} \mathbf{n} \cdot \nabla\bar{\kappa} dA \quad (13)$$

Additional boundary conditions are required to be specified for the non-local equivalent strain i.e.  $\bar{\kappa}$  or  $\mathbf{n} \cdot \nabla\bar{\kappa}$  have to be located in each point on the surface  $A$ . It is appropriate to set  $\mathbf{n} \cdot \nabla\bar{\kappa} = 0$  where  $\bar{\kappa}$  is not specified. Therefore, eqn.13 reduces to

$$\int_V m_\kappa \delta\bar{\kappa} \ddot{\bar{\kappa}} dV + \int_V \delta\bar{\kappa} \bar{\kappa} dV + \int_V c\nabla\delta\bar{\kappa} \cdot \nabla\bar{\kappa} dV = \int_V \delta\bar{\kappa} \kappa dV \quad (14)$$

Now, the weak form of the equilibrium of forces is given by

$$\int_V \delta u \cdot \ddot{u} \rho dV + \int_V \delta \varepsilon \cdot \sigma dV = \int_V \delta u \cdot b dV + \int_{A_i} \delta u \cdot t dV \tag{15}$$

with the acceleration normally represented as  $\ddot{u}$ , the symbol for specific mass  $\rho$ , the body forces indicated as  $b$  and traction  $t$ . Both the above weak form of equations (13,14) are discretised as

$$u = N_u \cdot U_I$$

$$\bar{k} = N_k \cdot \bar{k}_I \tag{16}$$

$N_u, N_k$  Are the shape functions and  $u_I, \bar{k}_I$  are the nodal point displacements and the nodal points non-local equivalent strain.

The spatial derivatives and their increments are given by

$$\varepsilon = B_u \cdot u_I, \quad d\varepsilon = B_u \cdot du_I, \quad \nabla \bar{k} = B_k \cdot \bar{k}_I,$$

$$d\nabla \bar{k} = B_k \cdot d\bar{k}_I \tag{17}$$

The test functions  $\delta u, \delta \bar{k}$  are discretised in a similar way. The eqns (16,17) together with weak forms (13,14) lead to

$$M \ddot{a} = f - r \tag{18}$$

With

$$a = \begin{pmatrix} u_I \\ \bar{k}_I \end{pmatrix} \tag{19}$$

non-local equivalent nodal strains below the nodal displacements and

$$M = \begin{bmatrix} M_u & 0 \\ 0 & M_k \end{bmatrix}, \quad r = \begin{pmatrix} r_u \\ r_k \end{pmatrix}, \quad f = \begin{pmatrix} f_u \\ f_k \end{pmatrix} \tag{20}$$

and

$$M_u = \int_V N_u^T \cdot N_u \rho dV, \quad M_k = \int_V N_k^T \cdot N_k \rho dV$$

$$r_u = \int_V B_u^T \cdot \sigma dV, \quad r_k = \int_V (N_k^T \bar{k} + B_k^T \cdot \nabla \bar{k} c) dV \quad \text{*Gradient enhanced damage modelling equations involve}$$

$$f_u = \int_V N_u^T \cdot b dV + \int_{A_i} N_u^T \cdot t dA, \quad f_k = \int_V N_k^T \cdot \kappa dV$$

solving for an additional degree of freedom, corresponding to  $\kappa$ .

Another method of rate dependent damage model is also implemented in which the damage evolution is delayed as below

The above damage parameter is substituted in equation 1 to get the stress. This value of dynamic damage is evaluated analytically and programmed.

An Analytical expression to evaluate the equation.

$$\omega_{dynamic} = \omega - \left( e^{-\left(\frac{\Delta t}{t_0}\right) * \omega_{dynamicprevious}} \right) + \left( \frac{(1 - \omega_{dynamic}) * t_0}{\Delta t} \right) * (\omega - \omega_{previous})$$

Is the relaxation time, given as a constant.

Where MAT represents the material id.

Alpha is residual stress considered as a constant. Beta is slope of the softening curve varying to correlate with experimental results. Kappa can be defined as the ratio of tensile strength of concrete to its elastic modulus. M is an inertial property.

- ELAW = 1 (exponential softening)  
2 (Ulrich combe paper)
- EQVSTRN = 1(mazaras criterion)  
2(modified von mises)  
3(Ulrich combe paper)

**Table 1: Damage parameters**

| Damage Parameters | Value |
|-------------------|-------|
| MAT               | 1     |
| Alpha             | 0.99  |
| Beta              | 1000  |
| Kappa             | 1E-04 |
| M                 | 1E-12 |
| ELAW              | 2     |
| EQVSTRN           | 3     |

**4. NUMERICAL APPROACH**

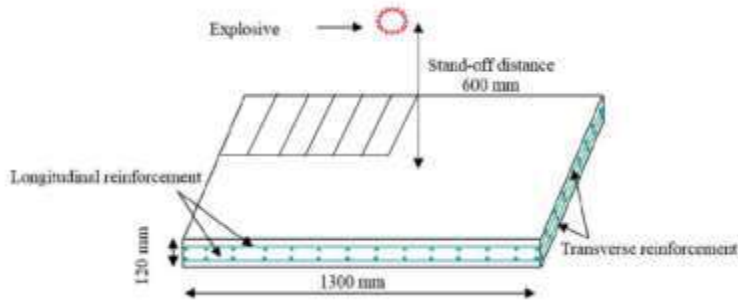
Fluidyn-MP 5.2.1 is used as a means of investigation in this study, which uses Lagrangian finite element methods to find the displacement corresponding to stress. Gradient-based damage model to locate the damage pattern as explained in detail above.

For much more detailed information on the numerical discretisation, the recommended article is Haeussler Combe et.al[12]. Fluidyn-MP uses skyline profile to store the stiffness and mass matrices and convected coordinate approach for higher displacements.

**5. NUMERICAL EXAMPLE**

The problem deals with the structural response of a slab of dimensions 1300mm\*1100mm\*120mm as shown in figure 4.1. The slab is reinforced at the top and bottom with 10mmsteel bars with 100mm and 200mm spacing in longitudinal and transverse directions respectively with 25mm cover. Due to symmetry, only a quarter portion of the slab as shown by the hatched portionFig.4.1 is modelled for analysis. The slab is fixed on its left edge. The concrete splice or part is modelled using hexahedral elements and the reinforcement as beam elements. The standoff distance is modelled as an ideal gas and the explosive as TNT with the weight of 2.09kg. The material properties are considered as per table [2,3 and 4]





**Figure 1 blast load over concrete slab setup**

**Table 2 Properties of Air and TNT**

| EOS for TNT              | JWL                     |
|--------------------------|-------------------------|
| Reference density of air | 1.255 kg/m <sup>3</sup> |
| EOS for air              | Ideal gas               |
| Reference density of TNT | 1630 kg/m <sup>3</sup>  |
| Gamma( $\gamma$ )        | 1.4                     |

**Table 3 Material Steel properties:**

|                     |                        |
|---------------------|------------------------|
| Young's Modulus (E) | 230 GPa                |
| Yield strength      | 560 Mpa                |
| Density ( $\rho$ )  | 7850 kg/m <sup>3</sup> |

**Table 4 the Concrete Properties C40**

|                           |                        |
|---------------------------|------------------------|
| Inertia parameter         | 1.0E-10 S <sup>2</sup> |
| Youngs modulus (E)        | 36 GPa                 |
| Poission's ratio( $\nu$ ) | 0.2                    |
| Density( $\rho$ )         | 2400kg/m <sup>3</sup>  |
| Damage exponent( $g_d$ )  | 2.0                    |
| Tensile strength          | 3.6 MPa                |
| Damage parameter( $e_d$ ) | 3.25E-03               |
| Damage parameter( $e_0$ ) | -6.77E-06              |
| Parameter $a_1$           | 3.1819                 |
| Parameter $a_2$           | -0.3419                |
| Parameter $a_3$           | 11.771                 |
| Parameter $a_4$           | 4.4077                 |



## 6. FINITE ELEMENT MODELLING

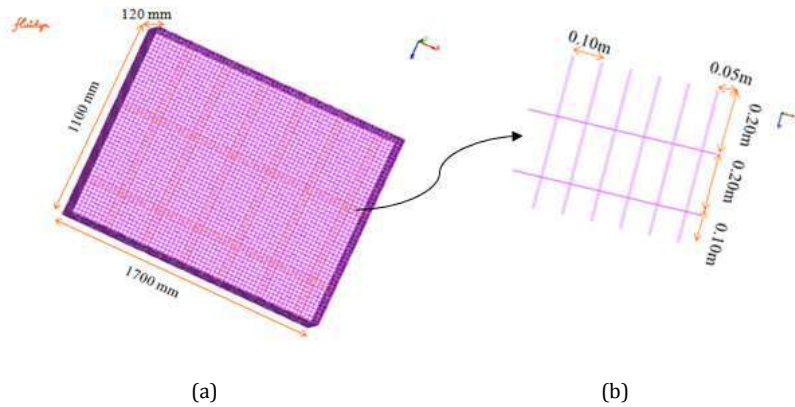


Figure 2 the finite element mesh (a) concrete with reinforcement (b) reinforcement detailing

The number of elements used is 39860 and the number of nodes is 43758. Transient explicit type of analysis is used for the case with Fluidyn-MP 5.2.1[14]. Hexa and beam elements were used for this case. Hexa elements to solve for the concrete portion and the beam elements will serve as a reinforcement

## 7. NUMERICAL RESULTS AND COMPARISON

The slab was subjected to blast and modelled only the quarter part of the slab due to its symmetry. Therefore the analyzed results will be applicable to the remaining portion of the slab. How symmetry gets satisfied is shown in validation cases. From the results, it can be terminated with that the symmetrical portion of the plate results veraciously compared with the full plate results. As the dimensions get diminished in the symmetrical notions the time essential for the solver gets reduced.

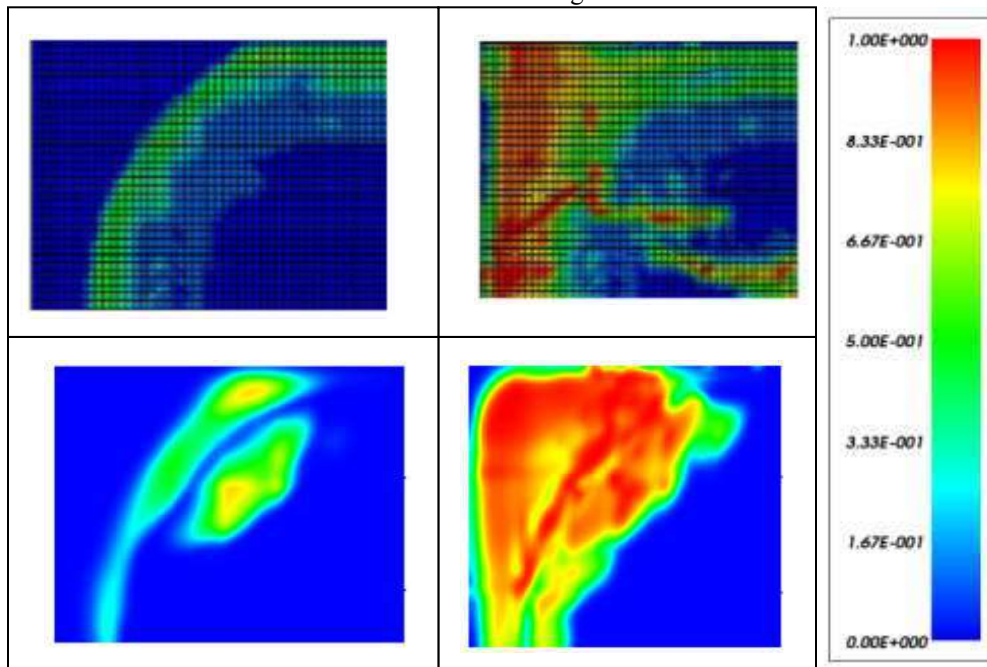
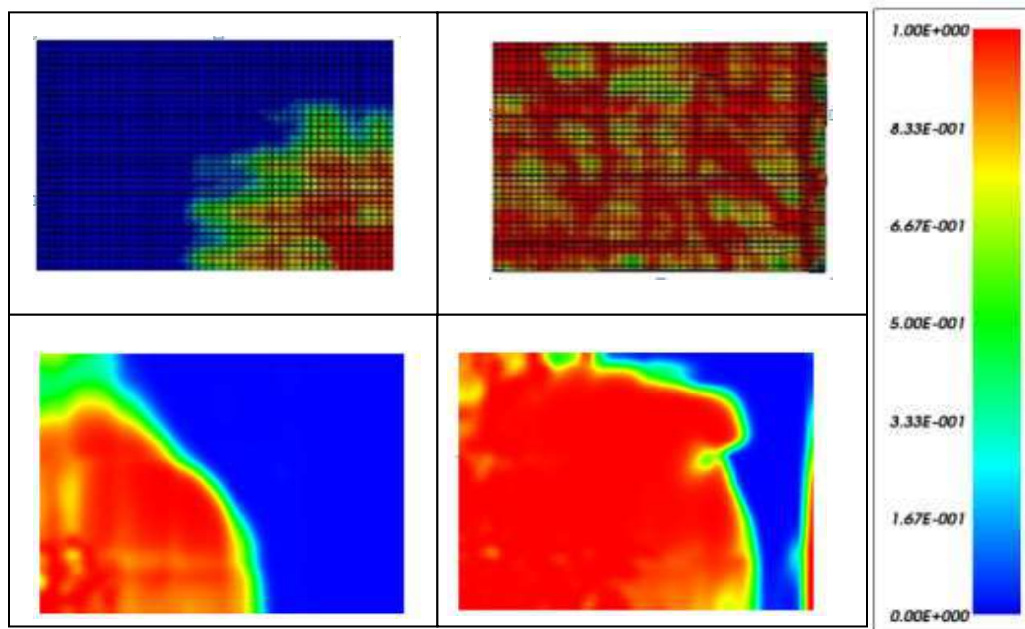


Figure 3 Damage at the top of the slab at a) 0.3ms (left) b) 0.4ms (right)



**Figure 4 Damage at the bottom of the slab at a) 0.3ms(left) b) 0.4ms(right)**

Figure 4.4 shows the damage pattern at the top of the slab at 0.3ms and 0.4 ms. Pictures at the top show the results of Lin et.al (2014)[13] and the pictures at the bottom show the results in the study disserted here using Fluidyn-MP[14]. Figure 4.5 shows the damage at the bottom of the slab at the same times and the reference pictures from Lin et.al (2014) [13] are shown on the left side.

The results are in good rapport with damage stockpiled at the support on top of the slab and with scattered at the centre at the bottom of the slab. However, the damage seems to be more scattered with the current rate dependent damage model and further parametric study with respect to the inertia parameter  $m_k$  is essential.

However, a one-to-one comparison cannot be made as the time intervals of plotting the results by Lin et al (2014)[13] are unknown and also the damage. Patterns are plotted in terms of fringe levels varying between 0 and 2 in Lin et al (2014)[13], in the case presented here, failure is plotted in terms of damage variable varying between 0 and 1. Only a qualitative collation is accomplished to manifest the position of failure.

## 8. CONCLUSIONS

- Above work can conclude the following things:
- A simple modelling approach based on continuum damage mechanics, gradient-based retarded inertia model and finite elements are effective to pragmatically anticipate intricate phenomena related to dynamic fracture of concrete.
- Gradient formulation for damage modelling derived from non-local theory is independent of mesh.
- For lower loading rates the crack propagation would be almost upright to the loading direction. At higher loading rates the crack branches out in two directions and can multiple directions for further loading rates.
- As it is a mesh insensitive, there won't a case with complex mesh which takes much time to solve algebraic equations.
- This code includes only concrete material; it can be extended to take up steel also.
- The code can be extended to take initial displacement (not only velocity) as boundary conditions.

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