



## **FUZZY QUEUES IN NONAGONAL FUZZY NUMBER THROUGH DSW ALGORITHM**

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### **ABSTRACT**

*This paper deals with a single server queuing model in nonagonal fuzzy numbers using  $\alpha$  – cut method through DSW (Dong, Shah & Wong) algorithm. The arrival rates and service rates are in nonagonal fuzzy numbers and also analyzed the performance measures are in fuzzy numbers using DSW algorithm. Numerical example shows the effectiveness of this model.*

**KEY WORDS:** Fuzzy queuing, Nonagonal fuzzy numbers,  $\alpha$ - cut, DSW algorithm, Interval analysis.

### **INTRODUCTION**

Operations research is the study of optimization technique and tools to solve problems in real life situations. Models are the essence of operations research which analyzing the behavior of the system to improve the performance.

Queuing models can be described as customers are arriving for service, waiting for service if service is not available immediately and leaving the system after being served. The most common queue disciplines are first come first service, last come first service and service in random order.

In this paper, we discussed about the single server queuing model and first come first served discipline using nonagonal fuzzy numbers under  $\alpha$ -cut through DSW algorithm.

**DEFINITIONS**

**Nonagonal fuzzy numbers**

A fuzzy numbers  $\tilde{A}$  is said to be a generalized nonagonal fuzzy number denoted by

$\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9)$  where  $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9$ ;  $k, w$  are real numbers such that  $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \leq a_6 \leq a_7 \leq a_8 \leq a_9$  and  $0 < k < w$  and its membership function given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < a_1 \\ \frac{1}{4} \left( \frac{x-a_1}{a_2-a_1} \right) & a_1 \leq x \leq a_2 \\ \frac{1}{4} + \frac{1}{4} \left( \frac{x-a_2}{a_3-a_2} \right) & a_2 \leq x \leq a_3 \\ \frac{1}{2} + \frac{1}{4} \left( \frac{x-a_3}{a_4-a_3} \right) & a_3 \leq x \leq a_4 \\ \frac{3}{4} + \frac{1}{4} \left( \frac{x-a_4}{a_5-a_4} \right) & a_4 \leq x \leq a_5 \\ 1 - \frac{1}{4} \left( \frac{x-a_5}{a_6-a_5} \right) & a_5 \leq x \leq a_6 \\ \frac{3}{4} - \frac{1}{4} \left( \frac{x-a_6}{a_7-a_6} \right) & a_6 \leq x \leq a_7 \\ \frac{1}{2} - \frac{1}{4} \left( \frac{x-a_7}{a_8-a_7} \right) & a_7 \leq x \leq a_8 \\ \frac{1}{4} \left( \frac{a_9-x}{a_9-a_8} \right) & a_8 \leq x \leq a_9 \\ 0 & x > a_9 \end{cases}$$

**Queuing formula**

$\lambda$ : The mean customers arrival rate,  $\mu$ : The mean service rate

The average number of customers in the system:  $L_s = \frac{\lambda}{\mu - \lambda}$

The average length of queue :  $L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$

The average waiting time in the queue:  $W_q = \frac{L_q}{\lambda}$

The average waiting time in the system:  $W_s = \frac{L_s}{\lambda}$

**STANDARD INTERVAL ANALYSIS ARITHMETIC**

Let  $I_1$  and  $I_2$  be two interval numbers defined by ordered pairs of real numbers with lower and upper bounds.

$I_1 = [a, b]$ ,  $a \leq b$  and  $I_2 = [c, d]$ ,  $c \leq d$

Define a general arithmetic property with the symbol  $*$  =  $[+, -, \times, \div]$  symbolically the operations

$$I_1 * I_2 = [a, b] * [c, d]$$

Represents another interval. The interval calculation depends on the magnitude and signs of the elements  $a, b, c, d$ .

$$[a, b] + [c, d] = [a+c, b+d]$$

$$[a, b] - [c, d] = [a-d, b-c]$$

$$[a, b] \times [c, d] = [\min(ac, ad, bc, bd), \max(ac, ad, bc, bd)]$$

$$[a, b] \div [c, d] = [a, b] \times \left[ \frac{1}{d}, \frac{1}{c} \right], \text{ provided } 0 \text{ does not belongs to } [c, d]$$

$$\alpha [a, b] = [\alpha a, \alpha b] \text{ for } \alpha > 0$$

$$\alpha [a, b] = [\alpha b, \alpha a] \text{ for } \alpha < 0$$

**DSW algorithm:**

DSW is one of the appropriate methods make use of intervals at various  $\alpha$ - cut levels in defining membership functions. It was the full  $\alpha$  – cut intervals in a standard interval analysis.

The DSW algorithm consists of the following steps:

1. Select a  $\alpha$ - cut value where  $0 \leq \alpha \leq 1$ .
2. Find the intervals in the input membership functions that correspond to this  $\alpha$ .

3. Using standard binary interval operations compute the interval for the output membership functions for the selected  $\alpha$  –cut level.
4. Repeat the steps 1 -3 for different values of  $\alpha$  to complete  $\alpha$  – cut representation of the solution.

**Numerical example**

Consider FM/ FM/ I queues where both arrival rate and service rate are nonagonal fuzzy numbers represented by  $\lambda^{\sim}=(1, 2, 3, 4, 5, 6, 7, 8, 9)$  and  $\mu^{\sim}=(13, 14, 15, 16, 17, 18, 19, 20, 21)$

The possibility of confidence interval as  $[1+ \alpha, 9- \alpha]$  and  $[13+ \alpha, 21- \alpha]$ .

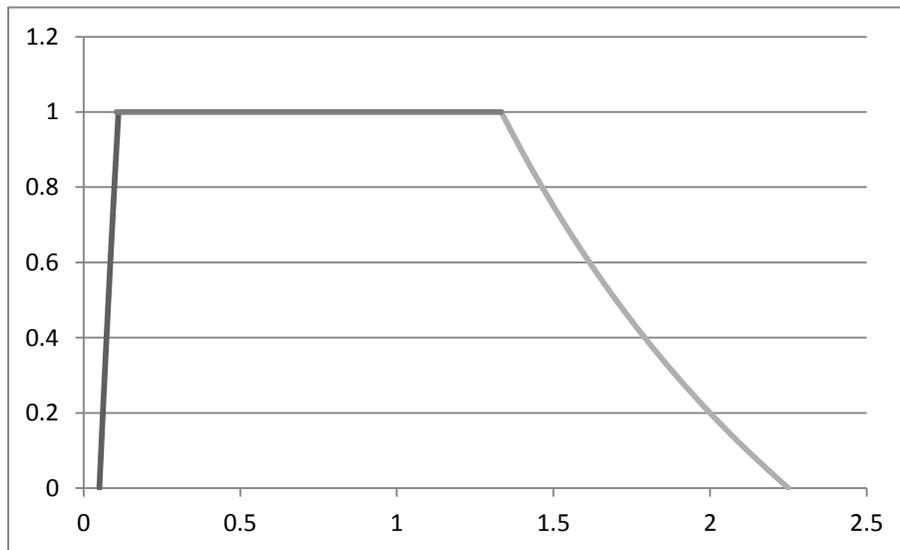
Let  $x = [1+ \alpha, 9- \alpha]$  and  $y = [13+ \alpha, 21- \alpha]$ .

$$L_s = \frac{x}{x-y}, W_s = \frac{1}{y-x}, L_q = \frac{x^2}{y(y-x)}, W_q = \frac{x}{y(y-x)}$$

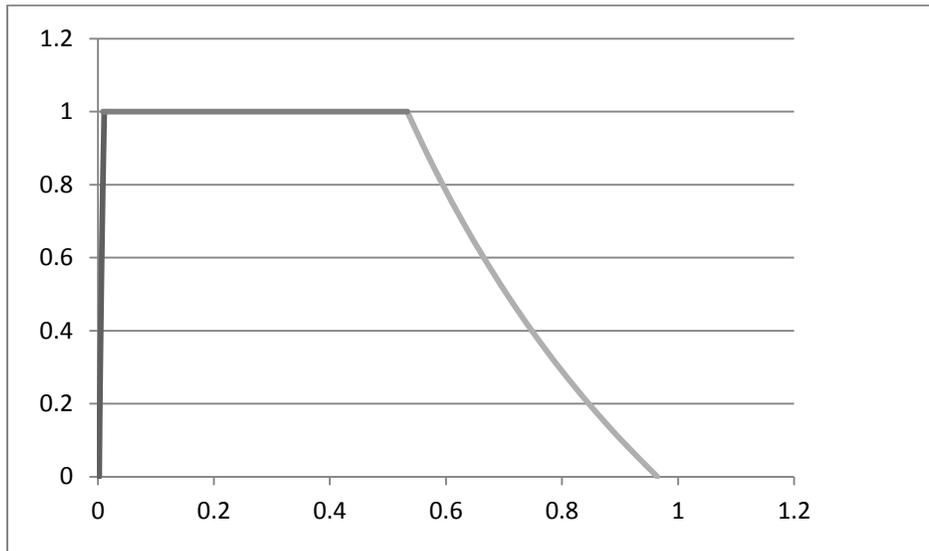
**Table:**

$\alpha$	$L_s$	$L_q$	$W_s$	$W_q$
0	[0.0500, 2.2500]	[0.0024, 0.9643]	[0.05, 0.25]	[0.0024, 0.1071]
0.1	[0.0556, 2.1190]	[0.0029,0.9024]	[0.0505, 0.2381]	[0.0027, 0.1014]
0.2	[0.0612, 2.0000]	[0.0035, 0.8462]	[0.0510, 0.2273]	[0.0029, 0.0962]
0.3	[0.0670, 1.8913]	[0.0042,0.7949]	[0.0515, 0.2174]	[0.0032, 0.0914]
0.4	[0.0729, 1.7917]	[0.0050, 0.7480]	[0.0521, 0.2083]	[0.0035, 0.0870]
0.5	[0.0789, 1.7000]	[0.0058, 0.7049]	[0.0526, 0.2000]	[0.0039, 0.0829]
0.6	[0.0851, 1.6154]	[0.0067, 0.6652]	[0.0532, 0.1923]	[0.0042, 0.0792]
0.7	[0.0914, 1.5370]	[0.0077, 0.6284]	[0.0538, 0.1852]	[0.0045, 0.0757]
0.8	[0.0978, 1.4643]	[0.0087, 0.5944]	[0.0543, 0.1786]	[0.0048, 0.0725]
0.9	[0.1044, 1.3966]	[0.0099, 0.5628]	[0.0549, 0.1724]	[0.0052, 0.0695]
1	[0.1111, 1.3333]	[0.0111, 0.5333]	[0.0556, 0.1667]	[0.0056, 0.0667]

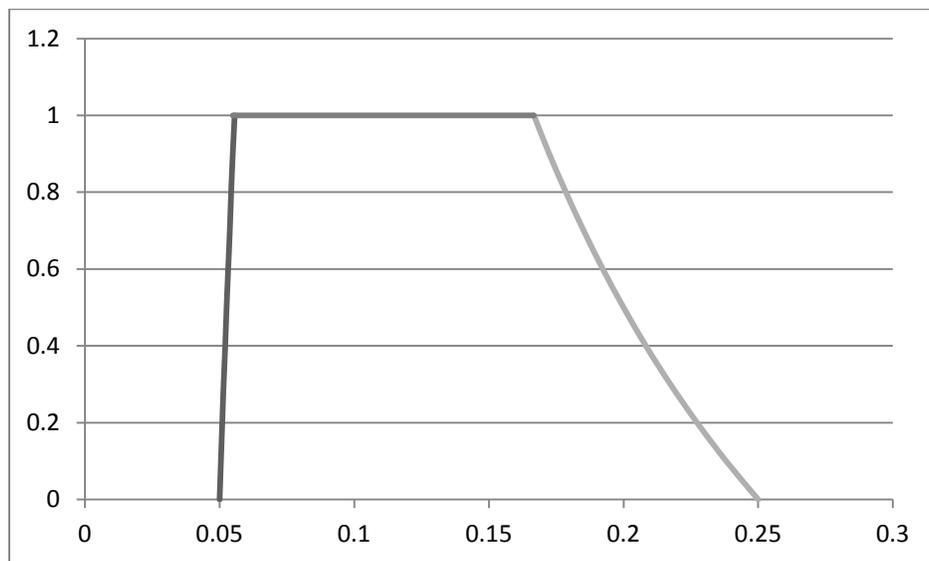
**Figure 1**  $L_s$



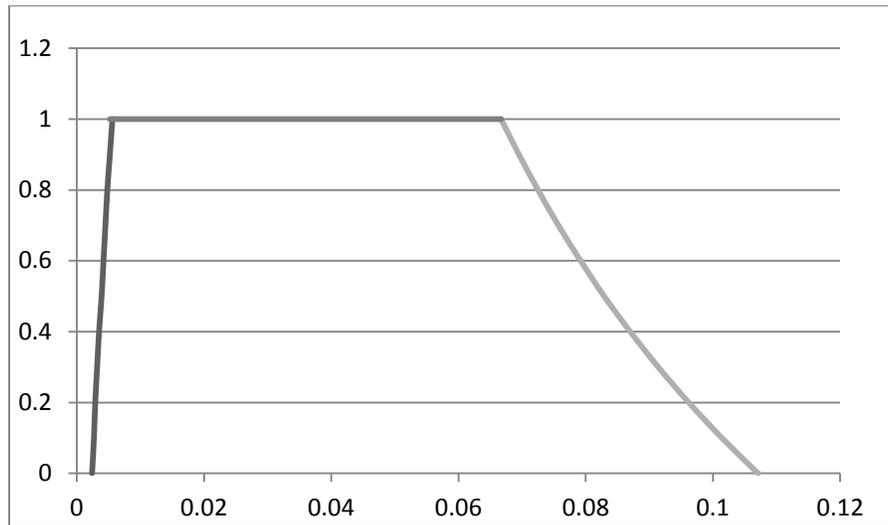
**Figure 2**  $L_q$



**Figure 3**  $W_s$



**Figure 4**  $W_q$



From table,

1. Expected number of customers in the system is 1.3333 and which is impossible falls outside the range [0.0500, 2.2500].
2. Expected number of customers in the queue is 0.5333 and which is impossible to falls outside the range [0.0024, 0.9643].
3. Average waiting time of a customer in the system is 0.1667 which is impossible to falls outside the range [0.05, 0.25].
4. Average waiting time of a customer in the system is 0.0667 which is impossible to falls outside the range [0.0024, 0.1071].

### CONCLUSION

In this paper we discussed the performance measures of single server queuing model in nonagonal fuzzy numbers. Analytical method and DSW algorithm are used to determine the performance measures such as expected number of customers in the system, expected number of customers in the queue waiting time in the system and waiting time in the queue which is also fuzzy. The numerical example shows the effectiveness of this approach.

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