



# ISSUES AND COUNTERMEASURES IN THE PROBLEM-SOLVING TRAINING FOR PRE-SERVICE MATHEMATICS TEACHERS

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## ABSTRACT

Through testing and interviews, this study investigates the problem-solving training of pre-service mathematics teachers during their university education. The findings indicate that their problem-solving abilities in mathematics have not improved effectively. The main issues identified include: the influence of secondary education, low professional identity towards the teaching profession, lack of emphasis on problem-solving training, and insufficient accumulation of theoretical knowledge in mathematical problem solving. To address these issues, the following countermeasures are proposed: universities should offer career planning courses related to the education sector to enhance pre-service mathematics teachers' professional identity and sense of belonging; pedagogy instructors should emphasize guiding pre-service teachers to study seminal works on problem-solving research to help them accumulate theoretical knowledge in problem solving; and pre-service mathematics teachers should strengthen self-analysis in problem-solving training, establish problem-solving portfolios, and improve their problem-solving skills.

**KEYWORDS:** Pre-service Mathematics Teachers, Problem-Solving Training, Mathematical Problem-Solving Ability

## 1. INTRODUCTION

Mathematical problem-solving is an essential process and method in learning mathematics. In the context of mathematics education in China, the prevailing view among most teachers is that mastering mathematics relies heavily on solving mathematical problems. Consequently, problem-solving occupies a critical position in mathematics instruction. Chinese students are required to solve a large number of problems as part of their mathematics learning, and Chinese mathematics teachers are known for their dedication to studying mathematical problems (Su, 2022). As future mathematics teachers at primary and secondary schools, pre-service mathematics teachers in higher education must enhance their mathematical problem-solving competence, which is essential for their future careers in teaching. In China, the situation regarding the problem-solving abilities of pre-service mathematics teachers has been noted by Cao (2014), who pointed out that the problem-solving levels of pre-service education students are generally low. He analyzed the reasons for their weak problem-solving skills and proposed corresponding improvement strategies and training models. Research by Wei (2005) and Tang (2006) also emphasized the importance of strengthening pre-service teachers' problem-solving abilities, providing effective methods to improve these skills. Furthermore, Wang and Xia (2012) summarized the main factors influencing the problem-solving ability of secondary school mathematics teachers and provided recommendations for improving the training of pre-service teachers, offering guidelines for measures to enhance their problem-solving abilities.

With the strengthening of the country's overall national power, the entry barriers for teachers in primary and secondary schools have gradually increased. This includes reforms in teacher certification and changes in teacher staffing systems, presenting challenges in employment for pre-service teachers. For graduates of local universities, the employment outlook is even more pessimistic. Recruitment for mathematics teachers often includes problem-solving tests as an important selection criterion. Many teacher education institutions have set up courses on primary and



secondary school problem-solving training to improve the problem-solving skills of pre-service mathematics teachers and enhance their competitiveness in the job market.

This paper aims to study the current status and ability levels of mathematical problem-solving training for pre-service mathematics teachers during their undergraduate studies, focusing on the factors affecting the effectiveness of problem-solving training, identifying existing problems, and proposing solutions.

## 2. CURRENT STATUS OF PROBLEM-SOLVING SKILLS AMONG PRE-SERVICE MATHEMATICS TEACHERS

### 2.1 Survey of Current Status

The study targets senior students majoring in Mathematics and Applied Mathematics in the teacher education program at the authors' university. It uses test questions to analyze their problem-solving abilities and selects representative students and pedagogy instructors for interviews to gain a deeper understanding of the main methods used in problem-solving training for pre-service mathematics teachers. The study also investigates the impact of teacher education courses on improving problem-solving skills.

#### 2.1.1 Testing Method

The test included questions of varying difficulty, such as elementary-to-junior high school math questions and questions from the high school entrance examination (detailed in the appendix). The aim was to assess the problem-solving ability level of pre-service mathematics teachers during their undergraduate years.

#### 2.1.2 Interview Method

Based on the test results and assessments of problem-solving training courses, thirty students were selected for interviews. These students represented different problem-solving ability levels: the top ten, middle ten, and bottom ten based on overall academic performance. The interviews aimed to gather in-depth insights into the current state and effectiveness of problem-solving training, and to identify any issues. To ensure the authenticity and validity of the research findings, the interviews with pre-service mathematics teachers were supplemented by discussions with pedagogy instructors from the Mathematics Department at the university.

### 2.2 Analysis of the Current Status

#### 2.2.1 Ability Analysis

An analysis of the test results revealed a significant gap between the overall problem-solving ability of pre-service mathematics teachers and the abilities expected of future primary and secondary school mathematics teachers. The distribution of scores in different levels is shown below:

Table 1: Distribution of Scores in the Elementary-to-Junior High School Test

Score Range	Below 60	60-70	70-80	80-90	90-100
Percentage	0.00%	10.29%	25.00%	38.24%	26.47%

The highest score in the elementary-to-junior high school test was 96, and the lowest score was 60. The average score was 82.60, with only 26.47% of the students scoring above 90. The scores were mainly concentrated between 70 and 90, with the overall performance being slightly above average. This indicates that there is considerable room for improvement in the problem-solving skills of pre-service mathematics teachers in elementary school mathematics.

Table 2: Distribution of Scores in the Junior High School Entrance Examination Test

Score Range	Below 72	72-84	84-96	96-108	108-120
Percentage	2.94%	20.59%	69.12%	7.35%	0.00%

The highest score in the junior high school entrance examination test was 100, and the lowest score was 68. The average score was 86.46, with a standard deviation of 6.37. The majority of students' scores were concentrated between 84 and 96, with relatively few students achieving high scores. This suggests that pre-service mathematics teachers' problem-solving abilities in junior high school mathematics are slightly below the expected level, not yet meeting the standards required for prospective teachers.



### 2.2.2 Analysis of the Causes

In order to understand the current state of problem-solving training for pre-service mathematics teachers and to further explore the reasons for their lower problem-solving abilities, interviews were conducted with pre-service teachers from different ability levels. These interviews aimed to investigate their problem-solving habits, their understanding of mathematical problem-solving theories, their views on the teacher education curriculum, and their professional identity as teachers, among other factors. Additionally, in-depth discussions were held with the faculty members responsible for teaching methods, to gain insights into the specific circumstances of problem-solving training from the perspective of the course implementers. The research identified several key reasons for the ineffective problem-solving training of pre-service mathematics teachers, which can be summarized as follows:

#### (1) Influence of Secondary Education

In secondary schools, students face significant pressure to perform well in entrance examinations such as the "Zhongkao" (high school entrance exam) and the "Gaokao" (college entrance exam). To achieve high scores, mathematics problem-solving is often reduced to "pattern recognition" and "memorization of techniques." The focus on repetitive problem-solving drills leads to the development of "problem-solving experts" who excel in solving specific types of problems. However, this method restricts the development of students' critical thinking skills. Students trained through mechanical methods struggle to form their own problem-solving strategies, and their understanding of mathematical knowledge and ideas remains superficial. After intensive training, when they enter university, most pre-service mathematics teachers feel relieved from the academic pressure and gradually lose enthusiasm for solving mathematical problems. As time passes, the problem-solving skills gained through rote practice are forgotten, with the speed of forgetting increasing over time. Furthermore, a lack of interest results in a diminished passion for problem-solving training. Consequently, it was observed that the problem-solving abilities of pre-service teachers generally declined during their university education.

#### (2) Low Professional Identity as Teachers

Interviews revealed that many pre-service mathematics teachers expressed uncertainty about pursuing a teaching career after graduation. Some realized during their internships that they were not suited for the education sector and lacked confidence in the teaching profession. Others hoped to continue their studies through graduate school, while some aimed to pursue civil service exams. The low level of professional recognition led to a lack of emphasis on problem-solving training, further contributing to the decline in their problem-solving abilities.

#### (3) Lack of Theoretical Knowledge on Problem-Solving

To some extent, the "drill and practice" method can help problem-solvers accumulate extensive problem-solving experience and develop a strong "mathematical intuition." However, theoretical guidance and the ability to generalize across problems are equally important. Interviews revealed that only a small number of pre-service mathematics teachers read resources beyond their course materials, such as literature on mathematics education theory and problem-solving techniques. The majority of pre-service teachers had limited knowledge of the theoretical underpinnings of mathematical problem-solving. Although they accumulated substantial problem-solving experience and improved their ability to solve problems, they lacked the theoretical foundation necessary to teach primary and secondary school students how to solve problems. This deficiency directly impacts the development of their professional teaching abilities.

Interviews also revealed that a lack of specialized training was another contributing factor to the ineffective development of problem-solving skills. Without long-term guidance from instructors, many pre-service mathematics teachers remained stuck in the problem-solving methods learned in middle or even elementary school. The absence of sufficient learning resources and support from their instructors meant that their problem-solving abilities were not effectively enhanced through the training process.

#### (4) Lack of Emphasis on Problem-Solving Training

The course *Mathematical Problem-Solving Training for Primary and Secondary Schools* is a practical course, typically scheduled for the second semester of the third year or the first semester of the fourth year. The main content and form of this course focus on solving problems in primary and secondary school mathematics. Through this course, the problem-solving abilities of pre-service mathematics teachers are somewhat improved. However, many students, having gone through the pressures of the *Zhongkao* (high school entrance exam) and *Gaokao* (college entrance exam), show a lack of interest in mathematical problem-solving. Additionally, since this course is a performance-based



assessment course, some pre-service teachers do not attach enough importance to the training, nor do they fully engage in the problem-solving exercises. Their lack of proper attitudes toward learning, weak initiative, and low expectations for their own learning have led to ineffective training outcomes, and their problem-solving abilities have not been significantly improved.

### **3. PRACTICAL STRATEGIES TO PROMOTE PROBLEM-SOLVING TRAINING FOR PRE-SERVICE MATHEMATICS TEACHERS**

#### **3.1 Institutions should implement career planning courses for the education sector to enhance pre-service mathematics teachers' professional identity and sense of belonging**

The professional identity of pre-service mathematics teachers in the education sector directly influences their interest and motivation towards teacher education courses. A higher sense of professional belonging and educational passion encourages pre-service teachers to actively improve their teacher quality, enhance their professional knowledge, and develop teaching skills. To acquire solid teaching fundamentals, they are more likely to consciously engage in problem-solving practice and improve their problem-solving abilities. Therefore, universities should introduce career planning courses early in the academic journey, focusing on career guidance and helping pre-service mathematics teachers with early professional development planning. These courses can strengthen their recognition of the education sector, boost their sense of responsibility and mission, and encourage them to proactively and consciously enhance their teaching fundamentals. From engaging in self-driven problem-solving exercises to improving their problem-solving abilities, pre-service teachers should gradually meet the required standards upon graduation.

#### **3.2 Teaching Methodology instructors should emphasize guiding pre-service mathematics teachers to study seminal problem-solving works and help them accumulate theoretical knowledge on problem-solving**

Instructors of teaching methodology courses generally guide pre-service mathematics teachers through the *Mathematical Problem-Solving Training for Primary and Secondary Schools* course. However, since this is a practical course with limited class hours, there is insufficient time to cover theoretical knowledge in depth. To avoid the tendency to "focus on practice and neglect theory," instructors should assign extracurricular reading tasks, encouraging pre-service teachers to independently study mathematical problem-solving theories. Key recommended texts include Luo Zengrui's *Introduction to Mathematical Problem-Solving* and *Theory and Practice of Solving Problems in Secondary School Mathematics*, as well as Pólya's three major works: *How to Solve It*, *Mathematics and Plausible Reasoning*, and *Mathematical Discovery*. These texts will help pre-service teachers build a strong theoretical foundation in problem-solving, ensuring that their practice is "guided by sound principles."

To ensure that pre-service teachers complete their extracurricular learning, instructors can require reading reflections or short essays as assignments to assess their understanding and engagement. Regular formative assessments can help track their progress.

With a foundation in educational theory, pre-service mathematics teachers should also connect their learning to real-life contexts. They should focus on constructing a coherent mathematical knowledge structure, exploring and developing a problem-solving methodology suited to their own needs. For example, they can apply Pólya's four-step problem-solving method—*Understand the problem*, *Devise a plan*, *Carry out the plan*, and *Review/Reflect*—to their own problem-solving practices. Through this process, pre-service teachers can identify common pitfalls and strategies for improvement at each step, thus aligning with Luo Zengrui's theoretical framework of "exploring how to solve problems through analyzing the process of problem-solving."

Finally, pre-service teachers should cultivate the habit of working on exploratory problems and accumulating problem-solving strategies. While real-world applications like studying Go or solving the Tower of Hanoi problem may not seem directly relevant, practicing such problems can enhance one's ability to solve more complex issues in everyday life. Therefore, engaging with exploratory problems is beneficial, as it trains mathematical thinking, improves mathematical skills, and enhances teaching abilities.

#### **3.3 Pre-Service Mathematics Teachers Should Strengthen Self-Analysis of Problem-Solving and Create Problem-Set Portfolios to Promote Improvement in Problem-Solving Skills**

According to Luo (2020), after going through the stages of "simple imitation" and "variant practice," problem-solvers enter the "spontaneous insight" phase. In this phase, insights often arise from intuition, manifesting as sudden clarity



or epiphany, which is often difficult to articulate and is more of a tacit learning process. This phase represents the self-discovery of *problem-solving strategies, enhanced problem-solving abilities, the formation of problem-solving techniques, and the refinement of problem-solving models*. It is a process of individual growth, where the person experiences substantial improvement through self-reflection (the generation of personal experience).

Currently, many pre-service mathematics teachers remain stuck in the first two stages—*imitation* and *practice*—and are unable to reach the "spontaneous insight" stage. Reaching this stage is considered an awakening in mathematical learning, where the learner realizes that problem-solving requires true understanding. Through constant problem-solving and self-reflection, learners gradually discover their own problem-solving methods, refine their strategies, and internalize approaches to tackle complex problems. This leads to significant, self-directed growth in their abilities.

According to constructivist learning theory, students actively build knowledge based on their prior experiences. Therefore, the extent to which a person analyzes a problem and applies various problem-solving methods varies greatly. This suggests that self-analysis of the problem-solving process is of great value for pre-service mathematics teachers. Problem-solving training for pre-service teachers should be an ongoing process throughout their undergraduate studies. The development of a *problem-set portfolio* serves as both a tool for self-reflection and a means for continuous improvement. This process allows pre-service teachers to evaluate their own progress, refine their abilities, and actively engage in their learning. It also provides an effective mechanism for cultivating their problem-solving skills (Wang, 2012). The creation of a problem-set portfolio motivates pre-service teachers to take an active role in their learning, engage in thoughtful self-reflection, and develop in a balanced and proactive manner. This is not only meaningful but also crucial for their professional growth.

#### 4. CONCLUSION

This study provides a comprehensive analysis of the current status and challenges faced by pre-service mathematics teachers in terms of their problem-solving abilities during their university education. Based on a survey involving both testing and interviews with fourth-year students in the Mathematics Teacher Education program, it highlights significant gaps in the mathematical problem-solving skills of pre-service teachers, as well as the contributing factors to these deficiencies. Despite some positive results from problem-solving courses, the findings indicate that overall, pre-service teachers do not reach the level of problem-solving competence expected for primary and secondary school educators. The causes of this underachievement are multifaceted, involving issues ranging from the effects of secondary education to a lack of professional identity, insufficient theoretical grounding, and limited emphasis on problem-solving training during university courses.

A key factor contributing to the weak problem-solving abilities of pre-service mathematics teachers is the influence of their secondary education experiences. Many students arrive at university with problem-solving strategies that were primarily honed through rote learning and exam-focused practices, which are often rigid and lack depth. This early educational background, while helping them to achieve high marks in exams, fails to nurture critical thinking or a deep understanding of mathematical concepts. Furthermore, the academic pressure of preparing for entrance exams often leads to a lack of engagement with more creative or exploratory problem-solving methods. As a result, by the time they enter university, many pre-service teachers have lost their enthusiasm for solving mathematical problems, and their skills deteriorate over time. This trend emphasizes the need for a shift in the way problem-solving is approached in both secondary schools and higher education, moving away from memorization towards more conceptual and creative problem-solving techniques.

Another significant barrier to the development of problem-solving skills among pre-service mathematics teachers is their low professional identity and uncertain career aspirations. The study found that many students do not view teaching as their long-term career, with some even expressing a desire to pursue graduate studies or alternative career paths such as civil service. This lack of professional commitment directly impacts their motivation to engage seriously with problem-solving training, as they do not perceive the importance of these skills for their future careers. Addressing this issue requires institutions to help students build a stronger professional identity early on in their academic journey. By integrating career planning courses and providing more career guidance, universities can help pre-service teachers recognize the value of developing robust problem-solving skills not only for their own growth but also as essential tools for effective teaching in the future.



The study also identified a critical gap in the theoretical knowledge that pre-service mathematics teachers possess regarding problem-solving methods. While many students gain substantial practical experience in solving mathematical problems, they often lack a strong theoretical foundation to support their problem-solving strategies. This deficiency in theoretical knowledge limits their ability to teach problem-solving effectively, as they are not familiar with the foundational theories that underpin mathematical thinking. To address this, it is crucial that pedagogy instructors encourage students to engage with seminal works on problem-solving theory, such as those by Pólya and Luo Zengrui. Integrating theoretical texts into the curriculum and assigning readings that emphasize the principles of problem-solving can help pre-service teachers build a deeper understanding of how to approach problems strategically, thereby improving both their own problem-solving abilities and their future teaching practices.

Furthermore, the study highlights the insufficient emphasis placed on problem-solving training within university curricula. The courses designed to improve the problem-solving skills of pre-service mathematics teachers are often treated as secondary to other subjects and lack the depth and focus required for meaningful improvement. Many students, having already been through intense training for the entrance exams, show little interest in engaging deeply with problem-solving exercises in university, particularly when these courses are not perceived as critical to their immediate academic success. To remedy this, it is important for universities to reframe problem-solving courses as essential components of teacher education and to foster an environment where students see these skills as fundamental to their professional development. Instructors should focus on creating a learning environment that encourages active participation, deep reflection, and self-directed problem-solving, thus helping students to see the long-term benefits of honing their problem-solving abilities.

In conclusion, the findings of this study suggest that improving the problem-solving abilities of pre-service mathematics teachers requires a comprehensive approach that addresses both personal and academic factors. By integrating career development courses, strengthening the theoretical foundation of problem-solving training, and fostering a more engaging and practical approach to problem-solving exercises, universities can better equip pre-service teachers with the skills necessary to succeed in their future careers. Furthermore, it is important to recognize that the development of problem-solving skills is a long-term process that extends beyond the classroom. By promoting continuous self-reflection and encouraging the creation of problem-solving portfolios, pre-service teachers can take ownership of their learning and progressively enhance their abilities. Ultimately, improving the problem-solving capabilities of pre-service mathematics teachers will not only benefit their professional development but will also contribute to the overall quality of mathematics education, as these teachers will be better prepared to guide future generations of students in their mathematical journeys.

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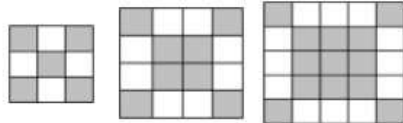


**Appendix 1:**

**Primary School Mathematics Test Paper**

**Section I: Multiple Choice Questions**

1. In the figure below, following the first three patterns, how many black squares are there in the fourth figure? ( )



A. 20    B. 25    C. 36

2. In the equation  $\frac{A}{4}$ , if  $A$  is a natural number not equal to zero, then when the value of  $A$  is ( ), the reciprocal of  $\frac{A}{4}$  is greater than itself.

A.  $A < 4$     B.  $A > 4$     C.  $A > 5$

3. The following is the score table for a mathematics exam taken by students of two classes in the sixth grade of a certain school (Unit: number of students).

	优秀	良好	及格	不及格
六 (1)	14	20	8	3
六 (2)	18	20	5	2

If obtaining excellent, good, or passing grades are all considered passing, then the correct statement is ( ).

A. Class 6(1) has a higher pass rate    B. Class 6(2) has a higher pass rate    C. None of the above statements are correct

4. Xiao Hua and Xiao Li spent the same amount of money to buy a box of apples. As a result, Xiao Hua received 8 kilograms, and Xiao Li received 12 kilograms. Therefore, Xiao Li needs to pay Xiao Hua 16 yuan. The unit price of the apples is ( ) yuan per kilogram.

A. 8    B. 4    C. 5

5. To cut a circular paper with an area of 12.56 square centimeters, the minimum area of a square paper required is ( ) square centimeters.

A. 12.56    B. 14    C. 16

**Section II: Problem Solving**

1. Qianjin Township plans to dig a water canal 300 meters long. They have already dug  $\frac{4}{5}$ . How many meters have been dug?



2. In 2018, the Zhaoqing Dinghu Mountain Scenic Area received a total of 3.92 million visitors. The number of visitors in the first half of the year accounted for  $\frac{3}{7}$  of the annual total, and the number of visitors in the third quarter was  $\frac{3}{4}$  of the first half-year's visitors. How many ten thousand visitors were received in the third quarter?
3. A certain automobile manufacturing plant produced 36,400 cars in the first half of the year, which is 3,900 more than the original plan. By what percentage did they exceed the production plan?
4. Xiao Ming is reading a book. On the first day, he read a portion, and the ratio of read pages to unread pages is 2:7. On the second day, he read 68 pages, and the ratio of read pages to unread pages becomes 4:5. How many pages are there in total in the book?
5. (Match as required)

Class 6-1 has completed 120 good deeds, \_\_\_\_\_, how many good deeds did Class 6-2 complete?

Class 6-1 has  $\frac{1}{3}$  more than Class 6-2  $120 \div \frac{1}{3}$

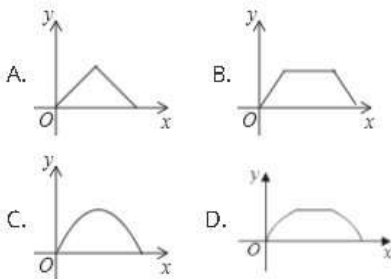
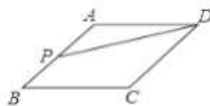
Class 6-1 has  $\frac{1}{3}$  less than Class 6-2  $120 \div (1 + \frac{1}{3})$

Class 6-1 is  $\frac{1}{3}$  of Class 6-2  $120 \div (1 - \frac{1}{3})$

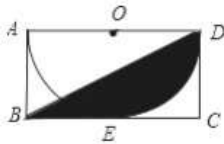
**Appendix 2:**

**Junior High School Test Paper**

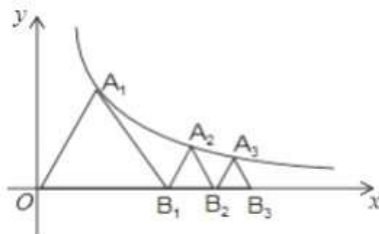
1. As shown in the figure, point  $P$  is a moving point on the side of rhombus  $ABCD$ . It starts from point  $A$  and moves uniformly along the path in  $A \rightarrow B \rightarrow C \rightarrow D$  to point  $D$ . Let the area of  $\triangle PAD$  be  $Y$ , and the movement time of point  $P$  is  $x$ . The graph of the function  $Y$  with respect to  $x$  is approximately ( ).



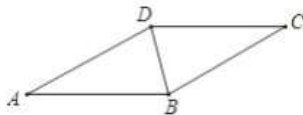
2. As shown in the figure, in rectangle  $ABCD$ ,  $BC = 4$ ,  $CD = 2$ . The semicircle  $AD$  with diameter as  $O$  is tangent to  $BC$  at point  $E$ . Connecting  $BD$ , the area of the shaded region is \_\_\_\_\_. (Result expressed in terms of  $\pi$ )



3. As shown in the figure, given an equilateral  $\triangle OA_1B_1$ , with vertex  $A_1$  lying on the hyperbola  $y = \frac{\sqrt{3}}{x} (x > 0)$ , the coordinates of point  $B_1$  are  $(2, 0)$ . Drawing a line through  $B_1$  intersecting the hyperbola at point  $A_2$ , and drawing a line through  $A_2$  intersecting the axis at point  $B_2$ , a second equilateral  $\triangle B_1A_2B_2$  is obtained. Continuing in this manner, ..., the coordinates of point  $B_6$  are .

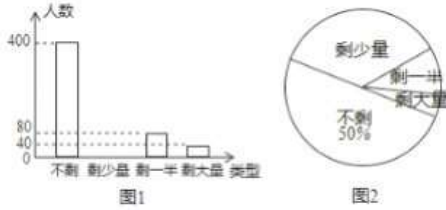


4. As shown in the figure,  $BD$  is a diagonal of rhombus  $ABCD$ .
- (1) Using ruler and compass construction, construct the perpendicular bisector  $EF$  of  $AB$ , with the foot being  $E$ . It intersects  $AD$  at  $F$ . (Construction steps are not required to be written; retain construction traces.)
  - (2) Under the conditions of part (1), connect  $BF$  and determine the measure of angle  $\angle DBF$ .

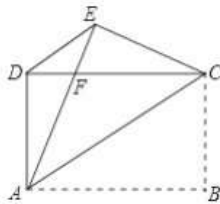


5. A certain company purchased a batch of  $A$ ,  $B$  model chips. The unit price of  $A$  model chips is 9 yuan less than that of  $B$  model chips. It is known that the number of  $A$  model chips purchased with 3,120 yuan is equal to the number of  $B$  model chips purchased with 4,200 yuan.
- (1) Determine the unit prices of  $A$ ,  $B$  and  $A$  model chips.
  - (2) If a total of 200 chips of both types were purchased, and the total cost was 6,280 yuan, how many  $A$  model chips were purchased?
6. A company's labor union conducted a survey on the "Weekly Workload Completion Status." A random sample of employees was surveyed regarding their remaining workload for the week, and the results were compiled into incomplete statistical charts as shown in Figure 1 and Figure 2.
- (1) The number of surveyed employees is .
  - (2) Complete the bar chart.

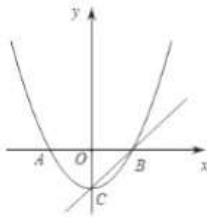
(3) If the company has 10,000 employees, estimate how many employees have a "small amount remaining" in their weekly workload.



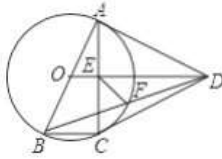
7. As shown in the figure, in rectangle  $ABCD$ ,  $AB > AD$ . Fold the rectangle along the diagonal  $AC$  so that point  $B$  falls on point  $E$ . The line  $AE$  intersects  $CD$  at point  $F$ . Connect  $DE$ .
- (1) Prove that  $\triangle ADE \cong \triangle CED$ .
  - (2) Prove that triangle  $\triangle DEF$  is isosceles.



8. As shown in the figure, the parabola  $y = ax^2 + b (a \neq 0)$  with vertex at  $C(0, -3)$  intersects the axis at points  $A, B$ . The line  $y = x + m$  passes through the vertex  $C$  and point  $B$ .
- (1) Determine the value of  $m$ .
  - (2) Determine the analytical expression of the function  $y = ax^2 + b (a \neq 0)$ .
  - (3) Does there exist a point  $M$  on the parabola such that  $\angle MCB = 15^\circ$ ? If it exists, find the coordinates of point  $M$ ; if not, please explain why.



9. As shown in the figure, in quadrilateral  $ABCD$ ,  $AB = AD = CD$ . The circle  $\odot O$  with diameter as  $AB$  passes through point  $C$ . Connect  $AC$ .  $OD$  intersects at point  $E$ .
- (1) Prove that  $OD \parallel BC$ .
  - (2) If  $\tan \angle ABC = 2$ , prove that  $DA$  is tangent to  $\odot O$ .
  - (3) Under the conditions of part (2), connecting  $BD$  intersects  $\odot O$  at point  $F$ . Connect  $EF$ . If  $BC = 1$ , determine the length of  $EF$ .



10. Given  $Rt\triangle OAB$ ,  $\angle OAB=90^\circ$ ,  $\angle ABO=30^\circ$ , and the hypotenuse  $OB=4$ , rotate  $Rt\triangle OAB$  clockwise by  $60^\circ$  around point  $O$  as shown in Figure 1, and connect  $BC$ .

(1) Fill in the blank:  $\angle OBC = \quad^\circ$ ;

(2) In Figure 1, connect  $AC$ , construct  $OP \perp AC$ , with the foot being  $P$ . Determine the length of  $OP$ .

(3) In Figure 2, points  $M$  and  $N$  simultaneously depart from point  $O$  and move along the edge  $\triangle OCB$ . Point  $M$  moves uniformly along the path  $O \rightarrow C \rightarrow B$ , and point  $N$  moves uniformly along the path  $O \rightarrow B \rightarrow C$ . When the two points meet, the movement stops. Given that the speed of point  $M$  is 1.5 units/second and the speed of point  $N$  is 1 unit/second, let the movement time be  $x$  seconds. The area of  $\triangle OMN$  is  $\mathcal{Y}$ . Determine the value of  $x$  when  $\mathcal{Y}$  reaches its maximum value. What is the maximum value?

